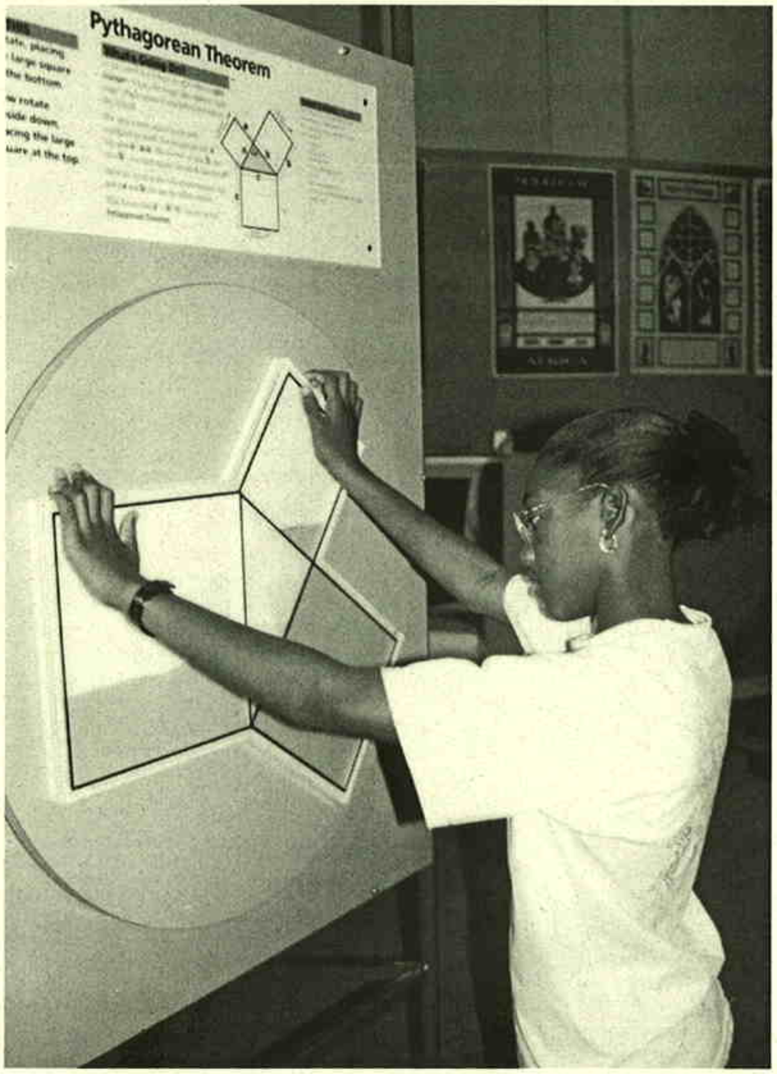
Mathematics in Science Centers



A report by
Andrea V. Anderson
for the
Association of Science-Technology Centers Incorporated

ASSOCIATION OF SCIENCE-TECHNOLOGY CENTERS INCORPORATED

Mathematics in Science Centers

By Andrea V. Anderson

**With assistance from Virginia Thompson,
Mathematics Consultant**

**Association of Science-Technology Centers Incorporated
Washington, D.C.**

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About ASTC

The Association of Science-Technology Centers Incorporated (ASTC) is an organization of science centers and museums dedicated to furthering the public understanding of science. ASTC encourages excellence and innovation in informal science learning by serving and linking its members worldwide and advancing their common goals. Through a variety of programs and services, ASTC provides professional development for the science center field, shares best practices, supports effective communication, strengthens the position of science centers within the community at large, fosters the creation of successful partnerships and collaborations, and promotes equity and diversity.

Founded in 1973, ASTC now has a membership that includes more than 580 science centers, museums, and related organizations in forty-three countries, making it the largest organization of interactive science centers in the world. Museums and science centers are sites for informal learning, places to discover, explore, and test new ideas about the natural world, and to develop positive attitudes about science. They function as unique resources for families, school children, teachers, and individual citizens. In increasing numbers, these institutions are also places where people of all ages, cultures, and education levels can find common ground. These diverse educational needs are also met by ASTC's many other member organizations, such as zoos, botanic gardens, nature centers, aquariums, planetariums and space theaters, natural history and children's museums.

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>
> I also thank DeAnna Banks Beane, who helped conceive the project, stimulate interest among science-center people, involve educators from the K-12 public-school system, and secure the funding. She has also been instrumental in visualizing the role science centers might have in helping children understand mathematics.

There were many people from science centers, universities, school districts, and from private and public organizations who contributed to the project. In particular, I wish to acknowledge the site-visit team members and to thank them for their time, insights and their contributions made to the report.

> First and deepest thanks go to Virginia Thompson, of Port Townsend, Washington. Although recently retired from the Lawrence Hall of Science, she agreed to serve as the mathematics consultant to this project. She visited each site, contributed her mathematics expertise when reviewing exhibits, programs, curriculum and other publications, and also reviewed the publication chapter by chapter, correcting both grammar and mathematics errors. She also provided the extended explication of the mathematics and grade-level connections for many of the exhibits.

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> Marlette Johnson, Scitrek, Atlanta, Georgia
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Finally, I thank the individuals who stepped forward from other sites and programs, such as the Houston Children's Museum, St. Louis Science Center, and the other institutions now offering mathematics, to share their work.

Andrea V. Anderson
June 20, 2001

Case Studies of Five Institutions

Background

The Association of Science-Technology Centers (ASTC), with funding support from the National Science Foundation (NSF), sponsored a study to ascertain the current status of mathematics in science centers and museums. The charge was to document examples of mathematics exhibits, programs, publications, and workshops in five case-study sites. In part the goal was to determine the feasibility of a mathematics initiative for science centers and their client schools.

A confluence of factors provided the impetus for the study. First came the results of the Third International Mathematics and Science Study (TIMSS), in which the mathematics scores for students in the United States dropped progressively from 4th grade to 8th grade and then finally finished near bottom in the 12th grade.

Second, the YouthALIVE![1] program sponsored a two-day meeting in Edmonton, Alberta, following the 1998 ASTC annual conference for science-center professionals interested in mathematics. At the conclusion of the sessions, participants recommended further efforts in elevating the presence of mathematics in science centers.

Finally, there was an increased interest on the part of governmental agencies, such as NSF, to find ways to improve mathematics achievement and a willingness to support efforts toward that end.

Small Grant for Exploratory Research (SGER)

Based on the factors mentioned above, ASTC obtained a Small Grant for Exploratory Research (SGER) from NSF to investigate mathematics in science centers and to publish the findings for the museum field and others. The purpose of the SGER project was to find and present models of museum-based mathematics programs and exhibits with an eye toward determining the feasibility of a mathematics initiative for science centers.

Method

Five science centers involved in the Edmonton meeting were selected for site-visits based on

- the willingness of the science center to share its work in mathematics;
- the science center's commitment to explore the potential for a fieldwide mathematics initiative; and

[1] YouthALIVE! (Youth Achievement through Learning, Involvement, Volunteering and Employment) is an ASTC initiative, supported by the Wallace-Readers Digest Funds, that utilizes educational enrichment and employment to integrate adolescents from underserved communities into science center operations.

- the insights to be gained regarding mathematics exhibits, programs, and/or materials and diversity of audiences[2].

The selected science centers were the Lawrence Hall of Science, the Exploratorium, the Fort Worth Museum of History and Science, the Pacific Science Center, and the Ann Arbor Hands-On Museum. A site-visit team—comprised of museum staff, one or two mathematicians and/or mathematics educators, and the project investigator—spent one or two days at each science center. Data were collected through interviews, observations, and reviews of exhibits, programs, and materials. Each team member participated in a group debriefing following the visit, and several prepared written comments.

Through the site visits, the team intended to learn about the kinds of programs, the types of mathematics, and targeted audiences. In addition, the team wanted to know what modes of delivery were used, whether the science center used or applied *The Principles and Standards for School Mathematics*[3] from the National Council of Teachers of Mathematics (NCTM) or state standards, and how effective the efforts were.

The following protocol of questions and concerns was used to guide the inquiry. However, the team was advised, prior to the site visit, that the current staff from the science center might be unable to answer all the questions.

- **Exhibits, programs, and materials description**—Identify the type, focus, and specific content of the exhibits, programs, and/or materials; the goals or intent behind the development of the program; and how explicit/implicit the mathematics is.
- **Development process**—Who was involved in the development (museum staff, school teachers, university faculty, community or business members, special interest groups)? How involved they were? If there had been a pilot effort, who selected the content and chose the medium?
- **Audience**—What age/grade level was intended? Was the audience intended or serendipitous? Was equity a consideration?
- **Role of standards**—The use of either school, district, state, or NCTM Standards.
- **Research or evaluation data**—Does research evidence support achievement of intent?
- **Lessons learned**—What insights should be shared when advising others about replicating the effort? Were these insights either unintended benefits or unintended consequences? What pitfalls can be avoided?
- **Funding sources**—How? Who? How much?

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[2] The sites represent a cross-section of museums, including science centers, a university-linked institution, and natural history and youth museums. A range of "products" was sought, including exhibits, teacher inservice and/or preservice programs, curriculum, trade books, web-based mathematics, and programs for school groups or children with parents.

[3] Unless otherwise identified as a state-level framework or standard, all references to Standards, or NCTM Standards, in this report will be in reference to this document.

Included in the Report

Case Studies

The case studies included in this document are intended to give the reader an image of how mathematics can be presented at a science center. For each site descriptions of the mathematics exhibits, programs, or materials are provided. Captioned photographs help the reader understand how the mathematics is portrayed. For some photographs, captions describe what the visitor does and learns in mathematics. Where possible, contextual information is provided about the science center to help the reader understand the philosophy and intent behind the development process.

The reader should be advised that these case studies are not an exhaustive catalog of mathematics offerings at each site. Also, while the corresponding NCTM Standard for each component is provided, the reader should be aware these linkages are after the fact. That is, the science center may or may not have relied on the Standards in the design and development of the exhibit, program, or materials.

Lessons Learned

At the end of each case study there is a section called Lessons Learned, which illuminates issues, ideas or opportunities that emerged during that particular site visit or during the reflections and debriefing of site visitors. The lessons learned helped frame the more general findings summarized in the next section.

Findings

The findings of the entire study are summarized below for the benefit of the reader. However, a more complete discussion of findings and emerging issues can be found beginning on page 99 following the case studies. The findings grow out of the lessons learned and the contributed insights of the site-visit team members, the professionals from each site, and the perspective of the authors.

- Science centers produce various kinds of mathematics exhibits, programs, and materials that serve multiple audiences.
- Science centers provide an important experiential component to the formal mathematics instruction provided by schools.
- Linking exhibits and programs to NCTM and/or state standards makes the science center a valuable resource for the K-12 school population.
- Highest-quality exhibits, programs, and materials come from "intentionality"—where the products were designed in concordance with a particular philosophical or theoretical stance and with commitment to achieving the intended outcome.
- The human resource—staff and advisors—is the science centers most valuable asset for engaging in mathematics conversations and experiences.
- Science centers choose to do mathematics exhibits, programs, and materials type based on what they already do well.

- The exhibits, programs, and materials are more successful and effective when relationships are built with end users as partners, and when partners are equal contributors to the outcomes.

- There is almost no research or evaluation of mathematics learning in science center exhibits, programs, or materials, although the potential is enormous.

Recommendations

Finally, the list of recommendations is reflective of ideas and suggestions offered by site-visit team members, as well as the professionals at the different sites. The reader is urged to review the discussion about each recommendation in the final section of this report. It is recommended that ASTC undertake the following:

- A capacity-building initiative that would enable science centers to offer more and better mathematics in more institutions, nationwide.
- A showcase (conference or workshop) of the current best mathematics exhibits, programs, and materials in science centers.
- Staff development for science-center professionals seeking to include mathematics in their exhibitions, program offerings, or materials.

- The creation of a working relationship with the National Council of Teachers of Mathematics and the American Mathematics Associations.

- The creation of a presence for mathematics by inviting mathematicians or mathematics-educators to address the science center field.

- Evaluation to find out if doing mathematics in science centers makes a difference in visitors' learning or attitudes about mathematics.

- An effort to reach out to universities, colleges, education organizations, and the K-12 school system as partners in achieving improved mathematics understanding by children and adults.

Lawrence Hall of Science

www.lhs.berkeley.edu/

Lawrence Hall of Science

Equity and Mathematics

The Lawrence Hall of Science, in Berkeley, California, has long been recognized for its high-quality curriculum products—earning Lawrence Hall a stellar national reputation among K-12 educators. The science center came to occupy this niche in part because it is located on the University of California's Berkeley (UC-Berkeley) campus and has been able to capitalize on this particular connection. However, Lawrence Hall, on its own merits, has been a magnet for intellectual and entrepreneurial talent, particularly in the field of mathematics education.

The purpose of this case study is primarily to describe the array of mathematics programs, products, and services at Lawrence Hall and to illustrate how mathematics came to be a major focus of a science center. The report describes the multiple external and internal forces that launched and sustained the initial mathematics effort and how new forces mediate the current programming. The long tradition of mathematics education places Lawrence Hall in an unusual position for providing leadership and/or material support for mathematics programming among science centers.

Mathematics programming began in the early 1970s, when a UC-Berkeley graduate student, Lucy Sells[4], noted that female students at the university were changing majors away from science because they were not prepared to do challenging mathematics. Alerted to the problem, Lawrence Hall staff member Nancy Kreinberg undertook her own investigation and found there were, indeed, fewer girls than boys coming to the science center to explore science and mathematics. Lawrence Hall then offered its first mathematics initiative, *Math for Girls and Other Problem Solvers*, which became in Kreinberg's words[5]:

> *...the seed for what has become a major focus at the Hall, encompassing workshops for teachers, conferences for adolescent women, and programs for working women.*

Equity concerns launched mathematics programs at the Hall. Over nearly three decades, Lawrence Hall staff developed equity-based, rich mathematics curriculum, exhibits, public programs, and enrichment offerings for families and children, mostly under the umbrella of the *EQUALS* programs. Mathematicians, mathematics educators, and equity advocates found a welcoming, supportive, and entrepreneurial environment at Lawrence Hall in which to flourish and to find out what works educationally and financially.

With *Math for Girls and Other Problem Solvers* as the foundation, Lawrence Hall developed new programs and products and forged a strong reputation for high-quality mathematics and equity programming. *Expanding Your Horizons, EQUALS*, and *FAMILY MATH* emerged during the 1970s and early '80s, but unlike other curricula from that era, *EQUALS*, and *FAMILY MATH* have

[4] Sells first introduced the concept of mathematics as the "critical filter" in her research report in *The Science Teacher*, 75 (1973): 28-29.

[5] Throughout this document, the quotes are intended not to be a verbatim transcription of what was said, but rather to capture the essence of what was meant. Any errors in fact or meaning are the author's and not the speaker's.

flourished and continue to evolve. They bring revenue into the science center, are used by thousands of teachers and are marketed across the nation and internationally.

Equity remains a driving element of Lawrence Hall's mathematics projects. Now as demographic changes rumble California, Lawrence Hall must adjust to an education reform agenda that sometimes moves laterally as much as forward. As external realities change, Executive Director Ian Carmichael has positioned Lawrence Hall to advance mathematics and equity by serving regional schools in new ways. Lawrence Hall is able to secure additional state funding (and visibility) for the science center, in part because of the strong equity thrust of earlier mathematics work. The *ACCESS* program, described on page 21, is the most recent iteration of the equity effort initiated by the *Math for Girls* project.

Challenges

Like many states, California faces the usual challenges to education: issues of teacher attrition, turnover, and inexperienced teachers. California also undertook a significant reform measure that lowered the teacher to student ratio to 1:20. Not surprisingly, in some circumstances, individuals were hired to teach without having the necessary credentials. California cannot ignore the issue of proper credentials, and so universities have been enlisted to help solve the problem.

California's affirmative action proposition rocked the state and the nation by disallowing such policies in governmental agencies. UC-Berkeley faced recruitment issues in which its traditional supportive aid could no longer be provided to capable minorities. Anticipating the demographic shift in California, wherein minorities have become the majority, UC-Berkeley had to find new mechanisms to bring people of color to the campus—to ensure that the future leadership is receiving the education they need to discharge work capably.

California has a significant, content-specific education challenge—reconciling the demands of the revised State Mathematics Framework with the vision and goals of the NCTM Standards. In a document of more than 300 pages, the new California framework articulates more specific content details than the NCTM Standards, and lowers the grade level for which the Standards should be achieved. For example, one goal established by the state framework is for all 8^{th} grade students to complete algebra. There is broad expectation that students will memorize mathematical algorithms and that teachers will not take the time to have students "construct" mathematical understanding. Funding from the state, which supports professional development, requires providers to ascribe to and support this view.

In that confluence of change-evoking forces, UC-Berkeley responded by adopting the Berkeley Pledge[6], which commits the university to reaching out to minorities in the K-12 program. Because Lawrence Hall had existing inservice mathematics programs for teachers working with

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[6] The Berkeley Pledge is "designed to fulfill the university's historic promise to maintain diversity while preserving excellence and to provide the best education to all of California's diverse student populations, particularly those whose opportunities are limited because they are financially or educationally disadvantaged."
For more information refer to http://www.chance.berkeley.edu/bpledge

minority and underserved populations, the University engaged the Hall as an active partner in achieving the outreach goals.

In mobilizing to respond to changing external realities, Lawrence Hall undertook a critical review of its operating structure. A new vision of Lawrence Hall—*being in service to schools and in support of school system changes that help students achieve*—places a stronger emphasis on reaching out to public education, especially those schools with significant minority populations. Staff now works more with public education offerings, doing customized workshops and programs, and less with curriculum development.

In the realm of mathematics, Lawrence Hall models a way to serve the community, elevate K-12 achievement, and retain its identity as an informal learning institution. Most of the high-caliber mathematics products from the past are still in use, and a powerful new relationship between the schools and the science center is emerging.

Mathematics at Lawrence Hall

Curriculum Products

Math for Girls and Other Problem Solvers, Lawrence Hall's first foray into mathematics, helped girls understand mathematics and develop positive attitudes toward the discipline. It was, in part the answer to these questions: How do we get girls involved with mathematics? How do we help girls see that mathematics is the foundation for many career choices—that without it, options decrease significantly?

At the early stages of *Math for Girls*, when young women were asked about career choices, their answers reflected societal images of the early 1970s. For example, girls considered glamour choices like modeling careers and could see little use for extend academic work in mathematics.[7] Because young teens, on their own and without proper guidance, often make choices with long-term negative consequences, especially in mathematics, doors to the future get closed prematurely. *Math for Girls* essentially said, "So, you are thinking about dropping math? Well, say goodbye to these career choices...."

The next phase developed in partnership with Mills College in Oakland. The program *Expanding Your Horizons,* cofounded by Lawrence Hall's Nancy Kreinberg and Lenore Blum of Mills College, augmented the career and mathematics connection. Teachers, together with middle and high school girls, attend a full-day conference, hearing from women who use mathematics daily in their work and participating in hands-on workshops.

Lawrence Hall's keystone program, *EQUALS,* is built on the success of working with a specific population and addresses the challenges experienced by underrepresented students—minorities and females—in achieving success in mathematics. *FAMILY MATH* came about when

[7] Early in the work on mathematics, EQUALS staff would ask students about what they were interested in. The responses at first revealed overwhelming interest in "glamorous" activities like modeling or being a sports superstar. By the end of the year, job choices were more balanced.

administrators and teachers requested strategies for family members to use in helping their children learn mathematics and develop "can do" attitudes. Customized teacher workshops provide curriculum and teaching strategies designed to help *all* children learn.

The concern for equity and achievement that framed the original direction for these programs, has not ameliorated. If anything, the need for accessible[8] mathematics is greater, not just for students, but for their teachers as well. *EQUALS, FAMILY MATH,* and the teacher professional development remain in tune with the changing demographics and populations using the products. Most of the curricula have been translated into Spanish, and a few are available in other languages as well.

EQUALS Investigations

The five *EQUALS Investigations* units were designed to approach mathematics from a problem-based model. For middle school grades, *EQUALS Investigations* promote a vision of mathematics in which students collaborate to solve interesting and somewhat complex problems. Each of the units takes one or more of the core strands from the NCTM Standards and sequences activities to challenge thinking and build concepts. Students are expected to collaborate because children learn in different ways and benefit from the thinking of others.

The units are rigorous and conceptually complex—reflecting current research on how the human brain is stimulated to solve problems (Sprenger, 1999; Wolfe, 1996). Students are visually or verbally clued into the critical mathematics connections they must make. Inherently, there is recognition that success may take a lot of time and thought, and teacher comments convey a "stick-with-it tone."

The activities within a unit build sequentially, and teacher commentaries in the margins of the page convey what really happens for students during each segment. Although within-unit activities are sequenced, *EQUALS* does not recommend a particular sequence of the units themselves. Each unit focuses on a particular concept from the middle school NCTM Standards. Table 1 on the following pages reflects the correspondence of units with the Standards.

[8] "Accessible" refers to an approach that is developmentally appropriate, connects to real world examples that make sense to students, and provides enough description or experience that concepts are understood. Symbolic representation and manipulation are secondary to conceptual understanding.

Table 1. Correspondence of *EQUALS Investigations* with NCTM Standards

NCTM Standards	EQUALS *Investigations*: Growth Patterns	Remote Rulers	Telling Someone Where to Go	Flea-Sized Surgeons	Scatter Matters
1. NUMBER AND OPERATION					
1.1 Understand numbers, ways of representing numbers, relationships among numbers and number systems	X	X	X	X	
1.2 Understand the meaning of operations and how they relate to each other	X	X	X	X	
1.3 Use computational tools and strategies fluently and estimate appropriately	X			X	
2. PATTERNS, FUNCTIONS, AND ALGEBRA					
2.1 Understand various types of patterns and functional relationship	X	X	X	X	X
2.2 Use symbolic forms to represent and analyze mathematical situations and structures	X	X		X	
2.3 Use mathematical models and analyze change in both real and abstract contexts	X	X	X	X	X
3. GEOMETRY AND SPATIAL SENSE					
3.1 Analyze characteristics and properties of two- and three-dimensional geometric objects			X	X	
3.2 Select and use different representational systems, including coordinate geometry and graph theory		X	X	X	X
3.3 Recognize the usefulness of transformations and symmetry in analyzing mathematical situations					
3.4 Use visualization and spatial reasoning to solve problems both within and outside of mathematics		X	X	X	
4. MEASUREMENT					
4.1 Understand attributes, units, and systems of measurement		X	X	X	
4.2 Apply a variety of techniques, tools, and formulas for determining measurements	X	X	X	X	
5. DATA ANALYSIS, STATISTICS, AND PROBABILITY					
5.1 Pose questions and collect, organize, and represent data to answer those questions	X	X	X	X	X
5.2 Interpret data using methods of exploratory data analysis		X	X	X	X
5.3 Develop and evaluate inferences, predictions, and arguments that are based on data		X	X		X
5.4 Understand and apply basic notions of chance and probability					

Table 1, *cont.* Correspondence of *EQUALS Investigations* with NCTM Standards

NCTM Standards	EQUALS *Investigations*: Growth Patterns	Remote Rulers	Telling Someone Where to Go	Flea-Sized Surgeons	Scatter Matters
6. PROBLEM SOLVING					
6.1 Work with problems	X	X	X	X	X
6.2 Formulate, represent, abstract and generalize	X	X	X		X
6.3 Use wide variety of strategies	X	X			
6.4 Reflect on their mathematical thinking	X	X	X	X	X
7. REASONING AND PROOF					
7.1 Recognize reasoning and proof	X		X		
7.2 Conjecture	X	X	X	X	X
7.3 Make mathematical arguments and proofs	X	X	X	X	X
7.4 Use various types of reasoning and methods of proof	X			X	
8. COMMUNICATION					
8.1 Communicate with others	X	X	X	X	X
8.2 Express mathematical ideas coherently	X	X	X	X	X
8.3 Consider the thinking and strategies of others	X	X	X	X	X
8.4 Use the language of mathematics	X	X	X	X	X
9. CONNECTIONS					
9.1 Use connections	X	X	X	X	X
9.2 Build mathematical ideas on one another	X	X	X	X	X
9.3 Recognize mathematics in contexts outside of mathematics	X	X	X	X	X
10. REPRESENTATION					
10.1 Use representations	X	X	X	X	X
10.2 Develop repertoire of mathematical representations		X			X
10.3 Use representations to model and interpret	X	X	X	X	

Funding for the development of *EQUALS Investigations* came from NSF to the Investigations Mathematics Curriculum Project at the Office of the President of the University of California. The funding allowed for two individuals from outside Lawrence Hall to author the publications, which were then reviewed, edited, and published under the *EQUALS* program umbrella. This organizational structure reflects the kind of collaboration increasingly found at Lawrence Hall.

Evaluation and research on Lawrence Hall's mathematics programs is incomplete. While controlled, refereed research was conducted for some of the individual projects or products during the development, for others it was not. There has been no research on the *EQUALS* program as a whole; however, the *EQUALS Investigations* materials were trial-tested in six local schools and then in schools across the nation. Outside researchers have studied the impacts of some of the materials, but the research is not comprehensive.

FAMILY MATH

Equity and access issues impelled the *EQUALS* team to consider alternative strategies to support mathematics achievement. Under the *EQUALS* program umbrella, other curricular products developed. Perhaps the best-known spin-off product has been the *FAMILY MATH* program, which seeks to involve more parents in their children's mathematics education. *FAMILY MATH* relies on the target parents' deep concern for their children and according to staff, depends on

> *the [target] parents' willingness to see that their children have a chance to do things that they [the adults] never had the opportunity to do.*

Sometimes parents, as much as their children, lack an understanding of what is required for a quality mathematics education. In a study funded by the National Action Council for Minorities in Engineering (NACME) (Leitman, Binns, & Unni, 1995), researchers found that 93 percent of parents do not know that the class choices middle school students are making have significance for their high school coursework and subsequent career options. Furthermore, the study noted that while minority students have reported a higher percentage of interest in being a scientist, they drop out of mathematics in greater numbers (Leitman et al., 1995). The *FAMILY MATH* program encourages students to take more mathematics and provides guidance for parents in counseling their children and helping them with math.

During *FAMILY MATH* programs, the staff reserves time for parents to talk with the instructor, to raise questions and find out the importance of specific topics and content. A particular concern for parents is ensuring that their children have facility with arithmetic. Parents tend to push for what they recognize as mathematics and typically push formal and abstract thinking too early. Parents want numbers—remediation in arithmetic—for their children, since that is their vision of mathematics. Yet, most parents' experience of this approach to mathematics—the manipulation of representational symbols—was odious and hateful. *FAMILY MATH* instructors work to broaden the parents' view of mathematics.

In many respects mathematics education has changed since the time parents were in school. Mostly this is a consequence of continued research on how children learn mathematics. *FAMILY MATH* instructors found that the most effective sessions begin with something familiar to the parents, such as number activities, but as lessons unfold they include a wide range of mathematics activities to entice the children. The challenge lies in presenting the activities so that parents recognize and understand the underlying mathematics content.

An activity may start with a number problem, where parents can see the mathematics, but the activity also has an element of play to entice the children. For example, one activity involves rolling dice. For children this is "luck," but the underlying mathematics is probability. Play helps build the understanding of mathematics and demonstrates that mathematics is more than numbers. Such sequenced learning helps parents construct a different vision of mathematics—one that is more fun and develops understanding. Comments from parents reflected the benefit of the *FAMILY MATH* experience for them:

> *I finally get what is going on in school. Before this, understanding what teachers wanted was hard.*
>
> *I get it now! I'm* finally *getting the mathematics I never got before.*

Sometimes the mathematics is not explicit, and instructors have the responsibility of helping parents see the connections and assuring them that they can support their child's learning, even if their own mathematics knowledge is limited. For example, the focus for *FAMILY MATH: The Middle Years* is algebra and number sense. Activities allow families access to algebraic reasoning, help family members see the mathematical thinking in the activities, and lay a foundation for formal algebra. Parents are taught to ask questions that direct attention to possible solutions: Can you tell me what the problem is about in your own words? Would it help to draw a diagram? Did you check your arithmetic?"

FAMILY MATH attempts to minimize barriers to mathematics learning and capitalize on unexpected opportunities. Language differences often mediate against student achievement and parental involvement. *FAMILY MATH* makes it possible for students to have a choice of language[9] for instruction, and lessons are modeled in two languages. Using the language most familiar to people minimizes this barrier to achievement.

Involving parents often provides an unexpected increase in instructional capacity of a lesson. A *FAMILY MATH* instructor described a "teachable moment" that generated from class participants:

> *In a class on measurement, we involved kids tracing hands or feet on paper, and cutting those "tools" out and using them to measure other things. One parent asked, "Since you are doing measurement, should my kid know how to use a ruler?" Other parents jumped in and shared stories and thoughts about useful connections and what they thought would be appropriate.*

FAMILY MATH was, and is, an extremely popular program. Lawrence Hall needed to respond to the demand for more inservice workshops. Using a trainer-of-trainers dissemination strategy, Lawrence Hall expanded the number of *FAMILY MATH* delivery sites to include museums, school

[9] *FAMILY MATH* is now available for Spanish-, Swedish-, and Chinese-speaking audiences. Portuguese is anticipated for the future.

districts, state and county departments of education, universities, and community organizations. The trainer-of-trainers model permits programmatic growth, as it maintains program integrity, builds a network of experienced and talented teachers, and provides Lawrence Hall with a point of contact within a community.

The *FAMILY MATH* books are available for sale through the Hall's web site and can be ordered individually, although their major use still occurs in workshops. There are currently more than 500,000 copies of *FAMILY MATH* for elementary and middle school in use across the nation. The publications are responsive to concerns heard from parents and educators. Since school culture is not always inviting to families, the newest publication, *FAMILY MATH: The Middle School Years*, includes a section on ways to help children make appropriate decisions and gives tools for parents to use in understanding the education system.

Mathematics in Public Programs

Public programming is usually the bread and butter of a science center. The draw of visitors, school groups, and families secures the financial foundation for most institutions. Lawrence Hall, like other science centers, makes use of available floor space for exhibitions and public events. However, Lawrence Hall makes public offerings work for them in other ways as well. In mathematics, three examples tell fascinating stories.

GEMS

Lawrence Hall pioneered engaging hands-on lessons for the visiting public. At some point, staff recognized that their public program activities had never been documented, so they set out to document and publish sets of activities generated for public programs, weekend enrichment activities, festivals, assemblies, and school programs. *Great Explorations in Math and Science (GEMS)* began when people started stringing the activities together to see which could be easily adapted for the classroom. *GEMS* is now probably the most widely used school enrichment curriculum.

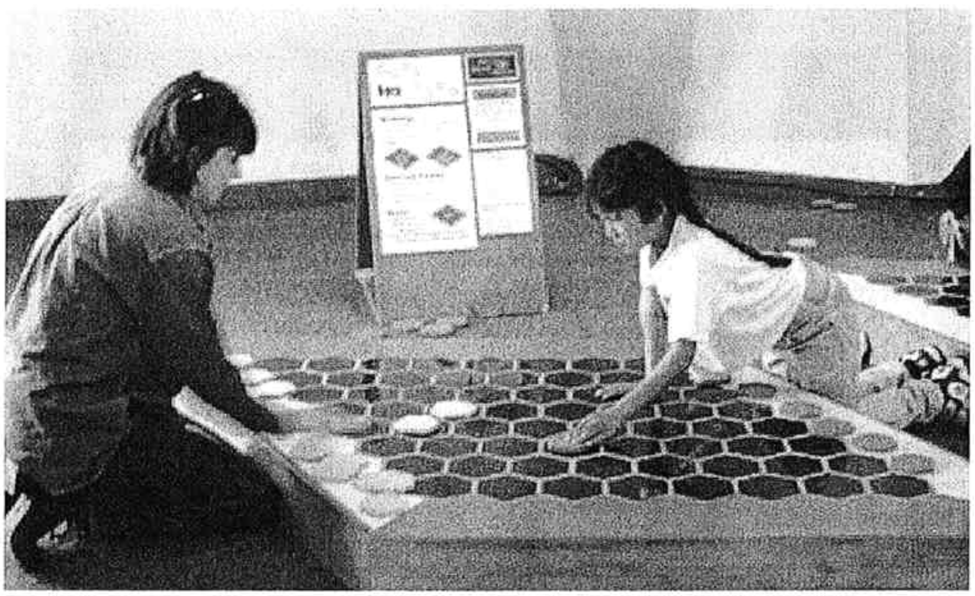
GEMS guides are organized by topic, with a conceptual theme or story line (e.g., a mystery) that ties each guide into a package. Activities are basically a collection of interactive tabletop stations that engage students in a variety of experiences but are not developmentally sequenced for learning. When *GEMS* is used in a classroom, students are encouraged to work with a partner and debrief with their teacher about the ideas learned.

The success of *GEMS* increased its usage in schools, because it brought hands-on learning to children and integrated mathematics and science meaningfully. *GEMS* now has 70 publications (60 teacher guides) and is taught in after-school classes, summer camps, and multisession courses. A large network supports delivery and distribution of *GEMS*, but like other Lawrence Hall products, requires a site to have a *GEMS* Associate, someone trained on the products. There are now more than 40 *GEMS* leadership sites—mostly located in teacher resource centers.

Although the GEMS books are extremely popular, their basic nature, the context of their origin, and their structural organization makes them appropriate as enrichment, but not as school curriculum. When queried about alignment with national standards, staff noted that the products stand philosophically with the NCTM Standards and the National Science Education Standards (NSES).

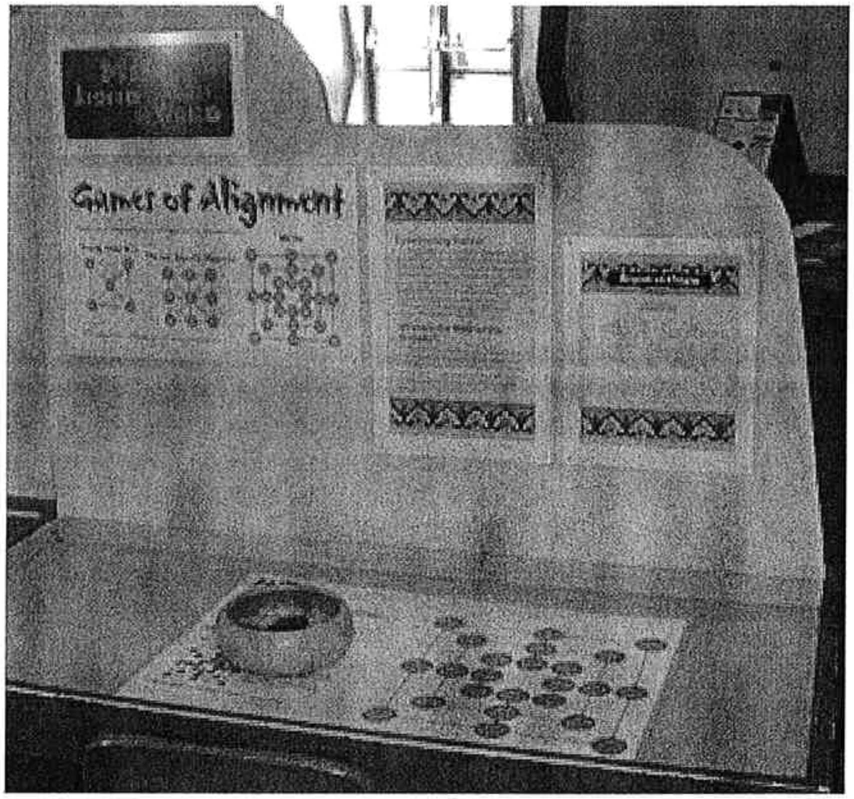
Math Around the World

Math Around the World started as classroom activities that were later gathered in a *GEMS* publication. Lawrence Hall's exhibit department took it on and created a number of mathematics exhibits that now occupy the space near the science center's entrance. The sequence of photos from *Math Around the World* shown in the following pages conveys the exhibition's potential and the interactive nature of its mathematics activities. The majority of the exhibits are games, which permit the visitor to have fun and learn mathematics content that aligns with the NCTM Standards. Also since the games are taken from the *GEMS* guide of the same name, visitors have the option of extending their play at home.



Hex is a recent invention created by Piet Hein and introduced at the Niels Bohr Institute for Theoretical Physics. The game requires the players to think offensively and defensively, as each person tries to create a pathway of tessellating pieces from one side of the board to the other. *Hex* corresponds to the NCTM Standard 3[10] – Geometry and Spatial Sense for using visualizations and spatial reasoning to solve problems. It also matches Standard 6 – Problem Solving, since players must use a wide variety of strategies to be successful.

[10] APPENDIX, page 116, provides the reader with an abbreviated chart of NCTM Standards developed by the author for use with this project. They are based on the draft version of the revised Standards. Any discrepancies noted between the chart and the now-published *Principles and Standards for School Mathematics,* 2nd Edition (2000) may reflect a revision of the draft or an error in transcription by the author.

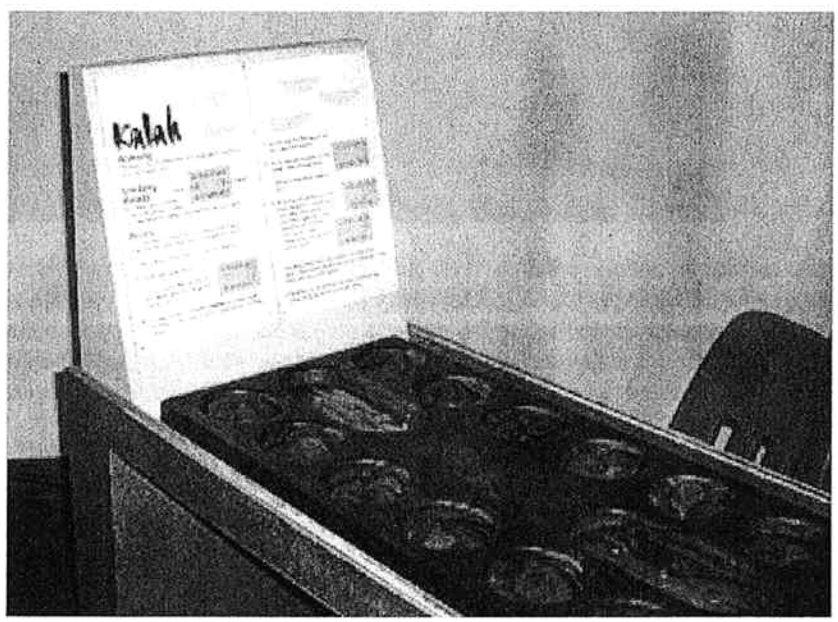


In the *Games of Alignment* exhibit shown above, visitors place colored markers onto a game board and try to block their opponent's pieces from moving. The exhibit addresses NCTM Standard 2 – Patterns, Functions and Algebra, and Standard 6 – Problem Solving.

The photograph to the right shows *Kalah*, the oldest-known continuously played game. Players move beans (or beads or stones) around the game board seeking to claim as many beans as possible.

Kalah addresses NCTM

- Standard 1 – Numbers and Operations;
- Standard 2 – Patterns, Functions and Algebra
- Standard 6 – Problem Solving

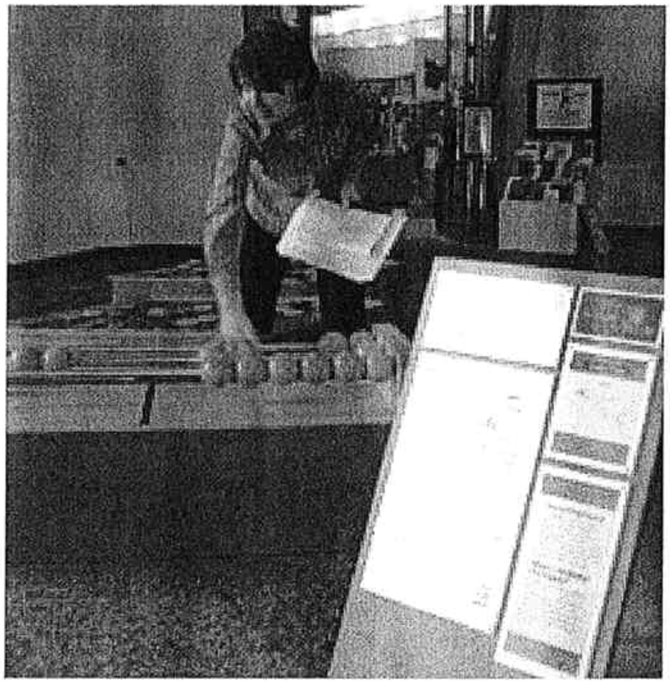


Game Sticks is a Native American game, sometimes used for gambling, in which the objective is to obtain all the counters. The game can be used to help students achieve several NCTM Standards:

- Standard 1 – Numbers and Operations
- Standard 2 – Patterns, Functions and Algebra,
- Standard 5 – Data Analysis, Statistics, and Probability
- Standard 6 – Problem Solving.

Magic Squares is a number game in which the object is to arrange numbers in cells so that they add up to the same total, regardless of whether the computation is done by row, by column, or diagonally.

There is an obvious link to Standard 1 – Numbers and Operations, but the game also introduces algebra in a fun way. Players must use logic and apply patterning skills.



Another game of strategy, *NIM* is used to introduce game theory (one of the branches of discrete mathematics), since the players actually need to construct a model to look at possible outcomes. The goal is to have a strategy that will let you always win by removing the last piece.

The NCTM Standards addressed by this challenging game are

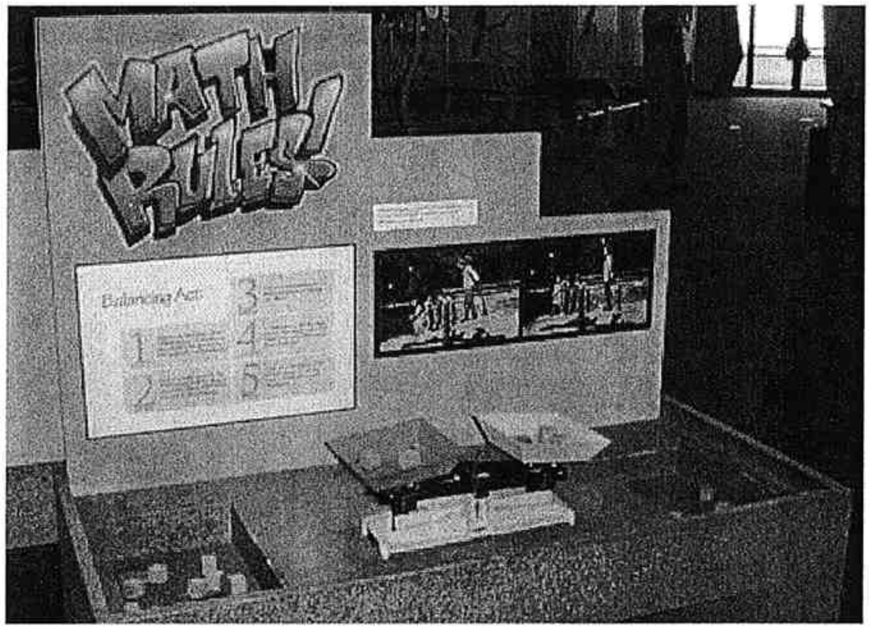
- Standard 1 – Number and Operation (item 1.1)
- Standard 2 – Patterns, Functions and Algebra (item 2.1)
- Standard 5 – Data Analysis, Statistics, and Probability (item 5.1)
- Standard 6 – Problem Solving (items 6.2 and 6.3)

The photograph to the left shows site-visit team member Melissa Thompson, from Pacific Science Center, exploring how NIM works.

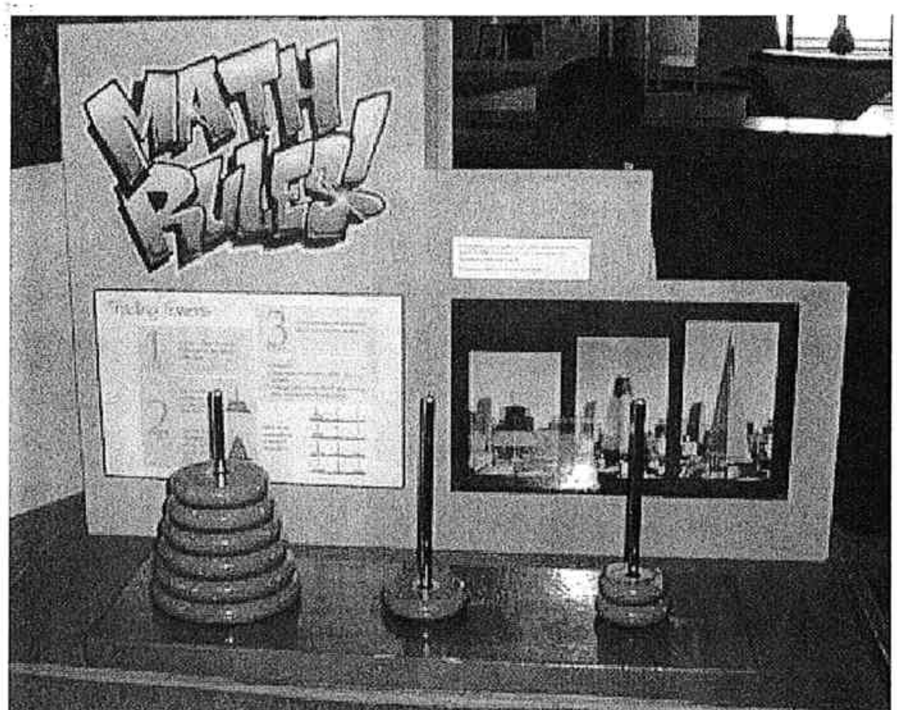
Young girl tries NIM on her own.

MATH RULES!

Another exhibit at Lawrence Hall focuses on mathematics. That exhibit, located in proximity to the *Math Around the World* exhibition, is called *Math Rules!* It does not have a teacher's guide, but seems as engaging as the games from *Math Around the World*. The activities in *MATH RULES!* were designed to reinforce concepts from NCTM Standard 2 – Patterns, Function, and Algebra. The following photographs display a few of the exhibits.

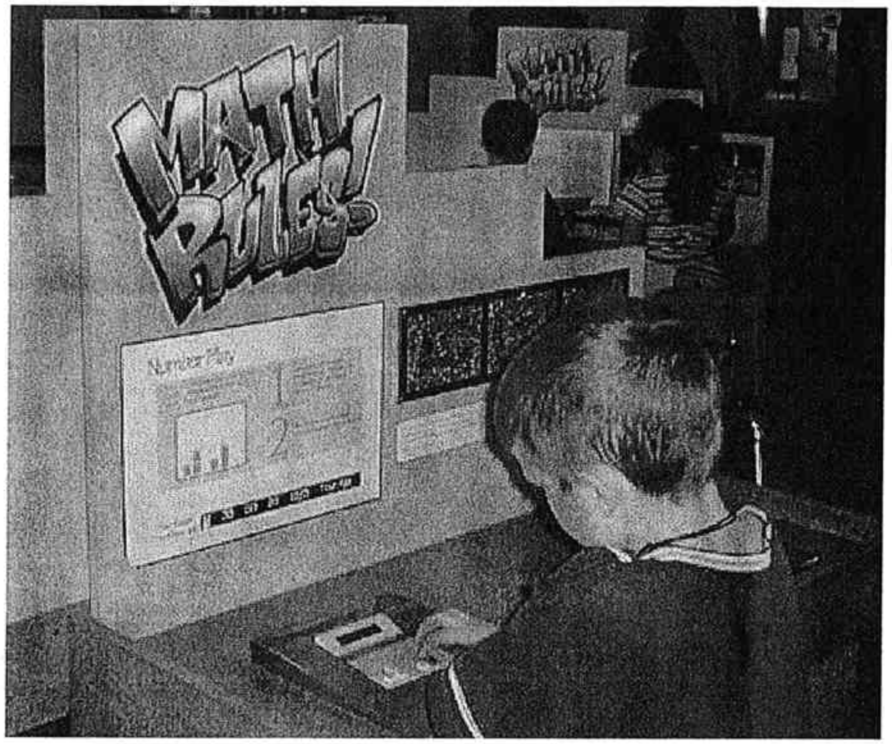


MATH RULES!
Balancing Act



MATH RULES!
Trading Towers

MATH RULES!
Number Play



MATH RULES!
Shape Up

Teacher Education in Mathematics

ACCESS (Alliance for Collaborative Change in School Systems)

Many of the curriculum products identified in the earlier sections rely on teacher workshops to disseminate the materials. In this section we focus on the most recent and probably most intensive teacher education program from Lawrence Hall. It is a model that requires significant outside funding, but it strongly illustrates Lawrence Hall's vision of working with schools—as reflected in the following sentiment expressed by staff:

> *Schools can't solve all the societal problems...but if we don't address mathematics achievement in school, then we simply replicate the gatekeepers.*

ACCESS is a professional development program for middle and high school teachers and a form of service learning for graduate students in mathematics. The program also provides professional development and technical assistance for school counselors and administrators. *ACCESS* aims to increase the minority and underrepresented populations in college—clearly sanctioned by the Berkeley Pledge. Structurally, *ACCESS* is a fluid set of relationships, involving Lawrence Hall staff, University of California faculty and graduate students, and selected local school districts.[11] Of course, as with any successful program, benefits accrue to all stakeholders, with each party getting something of value.

The participating districts contract with Lawrence Hall and assign a teacher to work with the *ACCESS* team. Monday through Thursday, the *ACCESS* teaching staff work full days within targeted schools. They offer workshops in the morning and spend the rest of the day working in specific classrooms. They also confer with individual teachers on specific problems encountered while teaching the subject.

Teachers in the schools receive broad support, not just in the area of mathematics content, but *ACCESS* also hopes to help teachers address equity and access issues for their students. The general approach is to improve teaching practices, including the use of new assessments. Most effort is focused on helping teachers examine curriculum, learn new teaching strategies, and meet the State Framework for Mathematics.

ACCESS coordinators from Lawrence Hall are assigned to four or five schools, where they spend up to 30 hours working with teachers, counselors, and administrators. On Fridays, *ACCESS* teachers and coordinators meet at Lawrence Hall to address concerns that emerged during the week. During the weekly sessions, the providers learn leadership skills, techniques for collaboration, and relationship building. At the same time, the meetings provide needed reality checks for the work, as well as time to remember the purpose and vision of the program.

ACCESS is also a peer-teaching program. *ACCESS* sends graduate students to work as instructional teaching assistants in the K-12 schools. The graduate students strive to make mathematics accessible for the middle and high school pupils, serving as tutors and classroom

[11] Oakland, Emery, San Francisco, and West Contra Costa Unified School Districts

aides. The graduate students, being close in age to the secondary school students, provide guidance and a support system for high school students considering career or college options. The college students are given training on how to work with school kids and are mentored throughout the program.

The graduate students benefit from this experience of teaching. In the competitive world of higher education, access to academic positions depends on presenting a portfolio of accomplishments. But while research remains the significant selection factor for positions in higher education, many new professors will be placed in teaching colleges rather than research-focused institutions. Some *ACCESS* graduate students enjoy being a member of a K-12 staff and decide to go back and get a teaching credential. As one graduate student observed:

> *The experience gained through ACCESS is unlike anything else my [peers] have. I have learned things not available to me from the rest of [my] program.*

Simply put, the graduate students benefit from knowing more about teaching and being able to point to successful experience in teaching.

When *ACCESS* employees are asked about the intent of their work, they respond "we want children to *think* about college, even if they don't enter." This answer is core to Lawrence Hall and is about equity, which is deeply embedded in Lawrence Hall's work in mathematics. Lawrence Hall and UC-Berkeley are committed to helping school districts understand and push for public school students' achievement in mathematics.

With *ACCESS*, Lawrence Hall created guidelines for placing students in mathematics courses. The program gives assistance to school counselors, helping them to understand the scope of mathematics and the sequence of coursework. As a result, students receive better advice, specific recommendations, and support in making appropriate choices. Administrators are included in addressing the socio-cultural issues that interfere with students' achievement. All school personnel are encouraged to consider changes in school culture and organizational structures that may create positive environments in which children might flourish.

Funding support for the program comes from a patchwork of sources. Districts contract with Lawrence Hall for the program, using funds provided by the state. State funding goes to the university to support graduate students and fund the teacher certification and professional development programming. Lawrence Hall collaborates with the education faculty, houses the *ACCESS* program, and monitors its implementation. Lawrence Hall receives Berkeley Pledge grant money to operate the program. Other funding comes from matching efforts. Recently one of the partner districts said they would fund one additional teacher (beyond the FTE count) to be in the teaching labs within each of the high schools.

Documentation of effectiveness is necessary for continued funding. There is an evaluation coordinator who is now documenting results. When *ACCESS* was launched in 1980 with the Oakland School District, there were only a few staff members serving a limited number of teachers. The program has grown to 15 full-time staff and now partners with three school districts, 27 elementary, middle, and high schools, and 15 collaborative programs, in addition to its core work.

Lessons Learned

There is much to be learned from an institution that has blazed a trail in mathematics. Lawrence Hall has produced a number of high-quality, widely accepted products. These products are well researched and grounded in best practices. Because they are aligned with NCTM Standards, they can serve as a framework by which other institutions can create programs and exhibits.

Mathematics expertise is important for developing quality products in this discipline.

Among the lessons learned, the importance of knowledgeable and talented staff is paramount. While science centers need not have mathematics PhDs on staff, they need access to competent people (staff or advisory) with extensive backgrounds in mathematics and mathematics education who can help translate content into experience. The depth of content understanding mathematicians bring to the enterprise ensures that the activities are more likely to develop deep ideas in mathematics and less likely to convey misconceptions. Because science centers attract scientists (and generalists) who may or may not be facile with mathematics learning, being assured of substantive, quality mathematics may depend on having committed outside advisors.

Lawrence Hall has access to UC-Berkeley faculty and academic talent from other Bay Area institutions, as well as having staff members with extensive mathematics experience. This struck the site-visit team as a significant advantage. Connections with UC faculty help to ensure content expertise and a strong advisory function. But Berkeley is not an isolated, university community; it also has a surrounding population that is intellectually supportive of research and development. Moreover, as a state agency Lawrence Hall can offer salaries that are attractive to the community pool of intellectual talent. A further benefit of being on the UC-Berkeley campus is having access to graduate students. Lawrence Hall is a built-in opportunity for students to do graduate research or to find employment before or after graduation.

Ultimately, Lawrence Hall has invested in high-caliber staff, which means that the Hall has deep capacity to deliver high-quality programs—a fact not lost on its federal, state, and corporate, funders. Grant-funded curriculum-development projects flourish at Lawrence Hall, in part because the Hall's purpose, supportive atmosphere, and available office space encourage collaborative exploration of educationally sound and financially supportive endeavors.

Using equity as a lens for mathematics seemed to lead to richer, more complete learning experiences.

Looking at mathematics through the lens of equity made for an effective strategy for developing discernibly richer learning experiences. As a gatekeeper discipline for many careers and college-program choices, mathematics must be accessible to young people, both in terms of opportunity and the quality of activities. Keeping their site firmly fixed on equity has provided Lawrence Hall the additional lens on mathematics that helped insure quality products that would work for all learners. The defining question seemed to be: What mathematics do children need to know in order to be successful? That is a different question from what mathematics can a science center offer and it seems to push toward deeper levels of learning.

Young people need to realize the importance of a mathematics education and acquire the conceptual understandings of higher-level mathematics. Breaching the barriers to higher-order mathematics is part of the overall reform effort in mathematics. Whether the barriers are deficits in skills or conceptual understandings, self-definition, or cultural perspectives and language obstacles, product research and development should target achievement for *all* students.

Start small, do what you already do well, and grow products from that foundation.

Lawrence Hall did not jump into large-scale mathematics programming. The process was more inquiry-like. An initial question launched *Math for Girls* and lead naturally to other equity questions and an expansion of offerings. It suggests that other science centers should start small and grow the programs based on the successes of their experiences. Of course, many of the Lawrence Hall programs offer workshops that could seed the endeavors at new science centers.

The philosophy of learner-centered experiences is a pervasive, subtle, and complex orientation at Lawrence Hall that might be defined as their "institutional culture." Current research on learning and best practices in mathematics is present in virtually every curricular product or workshop given by Lawrence Hall under the umbrella of the *EQUALS* program. A philosophical underpinning of constructivist learning permeates student activities, the directions given to teachers, and the exhibits. The flavor of reform is discernable within the activities and is more directly communicated in the Preface and Notes in *EQUALS Investigations*. It is this vision of mathematics and robust learning, developed over nearly 30 years, that makes Lawrence Hall of Science a flagship science center for mathematics.

Build partnerships with the formal system for support and for reaching new K-12 audiences.

Lawrence Hall made a decision to build connections with the formal education system—a decision that may or may not be attractive to other science centers. Curriculum development and programs targeting schools define the linkage, but the *ACCESS* program is the strongest and deepest connection with schools yet. It is a deep connection and a strong commitment, since it requires funding support for staff to be in school buildings, and a complicit agreement to abide by and acknowledge the NCTM and California Standards. In short, Lawrence Hall—in mathematics at least—has allied itself with the formal program and its fortunes will rise and fall with the schools.

On the positive side, the *ACCESS* relationships have brought in additional funds to Lawrence Hall through the University's Berkeley Pledge. There is a line item for the program in the state budget, with the money flowing to the science center through the university.

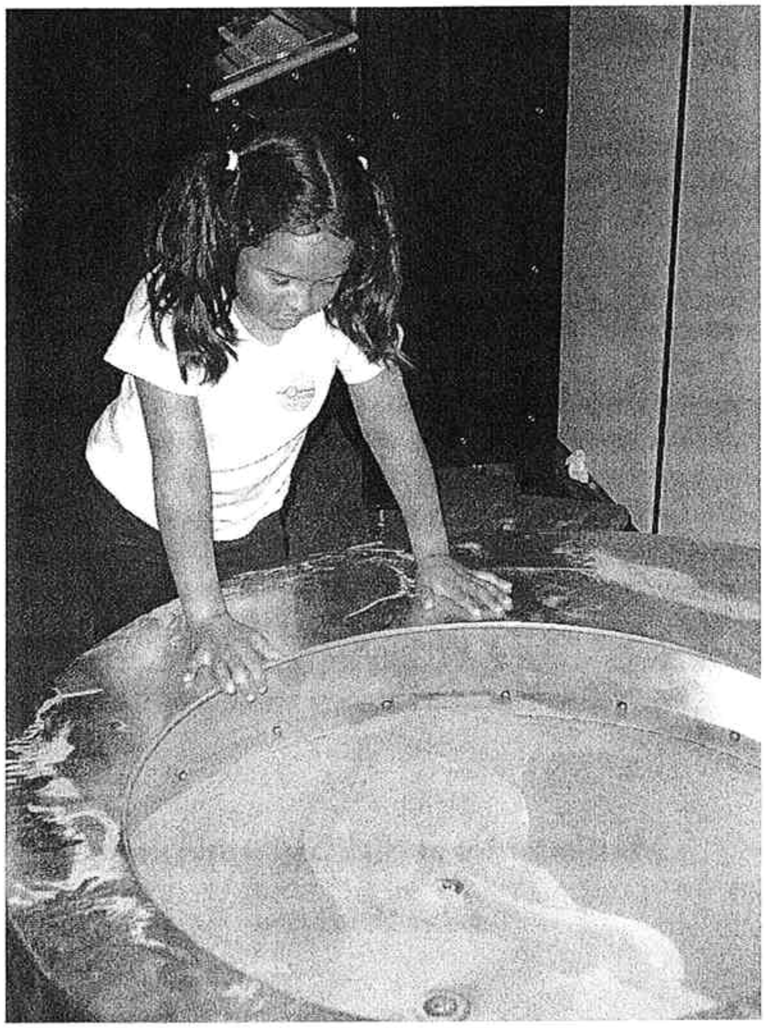
Lawrence Hall has been successful in mathematics in part because the staff listens to its constituents. They consider suggestions from teachers, parents, and caregivers and respond with creative, workable ideas. For example, *FAMILY MATH* is a direct outgrowth of concerns raised by teachers and principals who didn't know how best to involve families in the changing mathematics curriculum. At one point, a school district administrator said:

> *Parents are asking for ways to help their children at home, and my teachers are trying to answer that, but teachers are so busy they can't find the time and resources to help.*

By listening to these concerns, Lawrence Hall crafted an enormously successful program.

Since NCTM Standards and State standards may not match, a science center must consider how to reconcile the different expectations.

California is a microcosm of the political differences that find expression in how we expect teachers to function and children to behave. The recently legislated California mathematics framework is somewhat parallel with the NCTM Standards, but there is a distinction between them in how the material is delivered and used by students. The staff members, who have long-term professional affiliations with NCTM, must reconcile state expectations with NCTM philosophical perspectives. The same challenge may apply to other states as well.



The Exploratorium

www.exploratorium.edu

The Exploratorium

An Inquiry into Including Mathematics

The Exploratorium, in San Francisco, California, is not a typical case study for seeing what a science center has done in mathematics. Rather it illustrates how one science center, steeped in a "culture of inquiry," conceives ways to amplify or launch a mathematics presence. The questions the Exploratorium staff pursues, with regard to mathematics, are many of the same questions that the site-visit teams entertained throughout their visits. For example, the Exploratorium staff is considering how to make explicit the mathematics inherent in science exhibits.

Exhibit-based learning and personal inquiry—the legacy of Frank Oppenheimer—hold firm in the Exploratorium's self-definition and practice. Exhibits invite visitors "to do and notice" and education programs engage teachers in inquiry. The atmosphere of inquiry influences staff to imagine new approaches, ask "what if," and take risks in trying out ideas. The current work in mathematics is a work in progress—an inquiry—in which staff asks questions about best approaches, about using technology in new ways, and about planning for the future.

At the Exploratorium we found mathematics included in teacher workshops, publications, on the award-winning web site, and within exhibits. This case study describes how the Exploratorium has developed each component.

Mathematics at the Exploratorium

Teacher Education

The Teacher Institute

The Exploratorium launched the Teacher Institute for secondary-school physical science, open to teachers of science or mathematics in grades 6 to 12, during the 1980s. In part, the Institute was conceived as a way to draw teachers into the science center. By working with, and learning from the exhibits, teachers were more likely to bring their classes during the academic year. The National Science Foundation recognized the Teacher Institute as a credible professional development experience for science teachers and provided several years of ongoing funding. NSF continues to fund the Teacher Institute, which now also receives support from the State of California, Eisenhower, and foundations such as Hewlett and Noyce.

Mathematics first surfaced in the Teacher Institute when Thomas Humphrey, Exploratorium senior scientist, wondered about making the underlying mathematics in the physics exhibits more overt and explicit for teachers. Lori Lambertson, a teacher from a local district, joined the staff and began collaborating with Humphrey to explore the idea. They started by looking at existing exhibits with a mathematics lens and finding objects from which mathematical concepts could be discovered. They explained to teachers that *rather than focusing on the scientific principle in an exhibit, their task is to use the phenomena as a way to understand mathematics.*

This approach, according to Dennis Bartels, former Director of Teaching and Learning at the Exploratorium,

may be the easiest and most productive way for science centers to connect with mathematics. Applying a mathematics filter to existing exhibits and making the interpretation of the mathematics accessible to the visitor is relatively simple and cost-effective. New exhibits do not need to be constructed in order to do mathematics in the museum.

The Teacher Institute experiment with mathematics was captured in a video sequence called "Making a Difference," produced by PBS as part of its 1998 series *Life by the Numbers*. The video makes the point that science centers express a key principle of mathematics reform: relating mathematics to real-world examples. In a segment likely to connect with science center professionals, narrator Danny Glover makes an analogy between learning mathematics and learning a building trade. He notes that too frequently studying mathematics is like listening to lectures about hammers, screwdrivers, and the like:

At the end of the course you might know a lot about the tools, but still have no idea how to use them. This is how we have been teaching mathematics. Students memorize symbols and formulas, but they aren't taught the concepts that lie at the heart of mathematics. In short we have been teaching them tools, but not how to use them.

A real-world connection and authentic use of mathematical tools are core to the approach offered by the Teacher Institute. The teachers investigate real-world phenomena, and the process is reflective of the same intellectual engagement mathematicians and scientists employ. For the teachers, the real science phenomena, the mathematics, and the insights about teaching and learning make the Teacher Institute a powerful experience.

While teachers sometimes struggle with their own learning, they always reflect on how to translate these experiences into classroom lessons. To support them, the staff conducts Saturday workshops during the academic year to augment the experiences of the four-week Summer Institute. The teachers' conceptual understanding is improved, and the participants benefit from sharing classroom applications.

Over the course of 16 years, the Exploratorium has managed a fine balance between sustaining the popular Teacher Institute program and initiating new approaches (within the existing structure) that are attractive to funders. For example, the Teacher Institute now separates the physics and mathematics learning experiences into two summer sessions. The one for mathematics teachers is called "*Where's the Math*?"

Other changes have come in consequence of the decline in the number of qualified science and mathematics teachers staying in teaching. Turnover and decisions to leave teaching within the first three years of a career have serious ramifications for schools needing qualified staff. The Exploratorium has now modified the Teacher Institute to serve "new" science teachers—those in their first or second year of teaching—and involve alumni from prior institutes as mentors. Although the Exploratorium has launched this approach only in science education, it seems it could translate to mathematics professional development quite easily.

Mathematics on the Exploratorium web site

The Exploratorium created its award-winning web site[12] virtually at the birth of the World Wide Web. The fact that the Exploratorium has had a web presence since 1994 bespeaks institution support and strong leadership in this area. The web site is very comprehensive and has enormous potential to support mathematics in science centers. Since there are many directions one can go from the opening home page, it is advisable for readers to explore on their own. The following web site components were selected for this report because of their mathematics connections.

Snacks

Recasting Exploratorium experiences into student-centered, classroom-based lessons resulted in one of the Exploratorium's most teacher-friendly products, the Exploratorium *Science Snackbook.* This remarkable publication of teacher-created, small-scale exhibits for the classroom, known to staff as the *Snacks,* was the by-product of early Teacher Institutes. According to staff,

> *the* Snacks, *while not the "full-meal deal" of a science center visit, gives students a flavor of the informal learning experience.*

The *Snackbook* continued to evolve, included mathematics activities, and with the advent of the web site, the *Snacks* inevitably became a "cyber buffet" for teachers across the country who require their students to read and do activities from the *Exploratorium Snacks* page. Shown in Table 2 are modified versions[13] of activities from the *Math Snacks* page, with their corresponding NCTM Standards.

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[12] In 2000 the Exploratorium's web team earned ASTC's second annual Award for Innovation for their online achievement in developing www.exploratorium.edu.

[13] All images from the Exploratorium web site are proprietary. The science center's willingness to share and permit these images to be presented in modified format is appreciated.

Table 2. Activities on the Exploratorium's Math Snacks web page

SNACKS EXPLORATORIUM SNACKS

Snack Name	Description	NCTM Standards
Corner Reflector	The observer studies reflections in two mirrors set at right angles.	• Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.4) • Standard 4 – Measurement (item 4.2) • Standard 7 – Reasoning and Proof (item 7.2) • Standard 9 – Connections (items 9.1 and 9.3)
Inverse Square Law	Explore why the world gets dark so fast outside the circle of the campfire.	• Standard 1 – Numbers and Operations (items 1.1 and 1.2) • Standard 2 – Patterns and Functions (item 2.1) Standard 3 – Geometry and Spatial Sense (item 3.1) Standard 4 – Measurement (item 4.2) Standard 5 – Data Analysis and Statistics (item 5.1) Standard 6 – Problem Solving (item 6.1) Standard 7 – Reasoning and Proof (item 7.2) Standard 8 – Communication (item 8.1) Standard 9 – Connections (items 9.1 and 9.3)
Radioactive Decay Mode	Use coins to create a model for radiation	• Standard 1 – Numbers and Operations (item 1.1) • Standard 2 – Patterns and Functions (items 2.1, 2.2, and 2.3) • Standard 3 – Geometry and Spatial Sense (item 3.1) • Standard 4 – Measurement (item 4.2) • Standard 6 – Problem Solving (item 6.1) • • Standard 9 – Connections (item 9.1) • Standard 10 – Representation (item 10.3)
Solar Brightness	A photometer made by making grease spots on white paper can be used to compare the brightness of the sun to the brightness of a lamp. By finding a position at	• Standard 1 – Numbers and Operations (item 1.1) • Standard 2 – Patterns, Functions, and Algebra (items 2.1 and 2.2) • Standard 3 – Geometry and Spatial Sense (item 3.1) • Standard 4 – Measurement (item 4.2)

	which the sun is as bright as the lamp, the power output of the sun can be estimated.	• Standard 6 – Problem Solving (item 6.1) • Standard 9 – Connections (items 9.1, 9.2, and 9.3) • Standard 10 – Representation (item 10.3)
Spinning Blackboard	Create graceful loops and spirals by drawing on a spinning dish.	• Standard 2 – Patterns, Functions, and Algebra (item 2.1) • Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3) • Standard 5 – Data Analysis and Statistics (item 5.1) • Standard 6 – Problem Solving (item 6.1) • Standard 9 – Connections (item 9.3) • Standard 10 – Representation (item 10.3)
Spinning Cylinder	A spinning rod with a mark near one end is set rotating and spinning at the same time. Amidst the blur of the spinning cylinder, the mark appears three times, forming a stationary triangle.	• Standard 1 – Numbers and Operations (item 1.2) • Standard 2 – Patterns, Functions, and Algebra (items 2.1 and 2.2) • Standard 4 – Measurement (items 4.1 and 4.2) • Standard 5 – Data Analysis and Statistics (item 5.1) • Standard 6 – Problem Solving (item 6.2) • Standard 9 – Connections (items 9.1, 9.2, and 9.3) • Standard 10 – Representation (item 10.3)
Strange Attractor	The attraction and repulsion of magnets produces entrancing, unpredictable motion.	• Standard 2 – Patterns, Functions, and Algebra (item 2.1) • Standard 5 – Data Analysis and Statistics (item 5.1) • Standard 6 – Problem Solving (item 6.3) • Standard 9 – Connections (item 9.3)
Take It from the Top	Wooden blocks, stacked so the top block extends completely past the end of the bottom block, seem to defy gravity. A pattern emerges for how to place each block.	• Standard 2 – Patterns, Functions, and Algebra (items 2.1, 2.2 and 2.3) • Standard 5 – Data Analysis and Statistics (item 5.1)
Tired Weight	This Snack activity shows middle school and older students how to calculate the weight	• Standard 2 – Patterns, Functions, and Algebra (item 2.2) • Standard 4 – Measurement (items 4.1 and 4.2)

	of a car by finding the surface area of a tire footprint. It reinforces the understanding of surface area and units used in the measurement of area and pressure. It also provides an example of dimensional analysis, where measurements units are treated as parts of an equation.	• Standard 5 – Data Analysis and Statistics (item 5.1) • Standard 6 – Problem Solving (item 6.3) • Standard 9 – Connections (items 9.1, 9.2, and 9.3) • Standard 10 – Representation (item 10.3)
Vector Toys	Components of force cause this robot toy to walk and stop at just the right time.	• Standard 1 – Numbers and Operations (items 1.1, 1.2, and 1.3) • Standard 2 – Patterns, Functions, and Algebra (item 2.2) • Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.2) • Standard 4 – Measurement (items 4.1 and 4.2) • Standard 5 – Data Analysis and Statistics (item 5.1) • Standard 6 – Problem Solving (item 6.2) • Standard 9 – Connections (items 9.1, 9.2, and 9.3) • Standard 10 – Representation (item 10.3)

Other web site links to mathematics

The Exploratorium has linked other mathematics web sites to its own web site. A few of the mathematics sites are listed below. Check out the Exploratorium's Cool Sites web page for the rest.

Table 3. Exploratorium web site links to mathematics

Exploremath.com	http://www.exploremath.com/
Cool Math 4 Kids	http://www.coolmath4kids.com/
Figure This! Math Challenges for Families	http://www.figurethis.org/index40.htm
The Abacus: The Art of Calculating with Beads	http://www.ee.ryerson.ca:8080/%7Eelf/abacus/
Native American Geometry	http://www.earthmeasure.com/
The Math Forum	http://forum.swarthmore.edu/
The KnotPlot Site	http://www.cut-the-knot.com/content.html
Julia and Mandelbrot Set Explorer	http://aleph0.clarku.edu/~djoyce/julia/explorer.html

Science Explorer

Another section of the Exploratorium web site that addresses mathematics is the *Science Explorer,* based on the Exploratorium's award-winning publication of the same name.

In the example below, "Geodesic Gumdrops" (extracted and modified from the web site), students explore geometric shapes with simple materials and learn about the properties of squares and triangles. The corresponding NCTM Standards are:

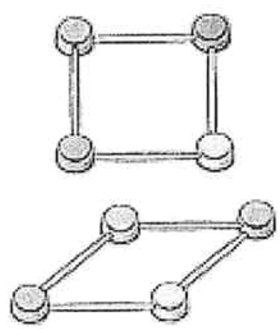
- Standard 3 – Geometry and Spatial Sense (item 3.1)
- Standard 5 – Data Analysis, Statistics and Probability (item 5.1)
- Standard 6 – Problem Solving, (item 6.1)
- Standard 9 – Connections (item 9.3).

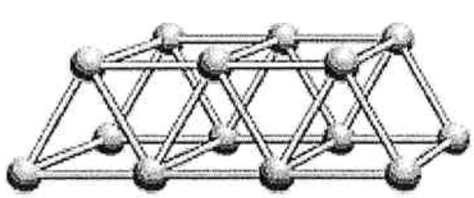
Geodesic Gumdrops

Make amazing architecture with candy and toothpicks.

What's Going On?	**What's the big deal about triangles?**

As you've probably already discovered, squares collapse easily under compression. Four toothpicks joined in a square tend to collapse by giving way at their joints, their weakest points. A square can fold into a diamond, like this:



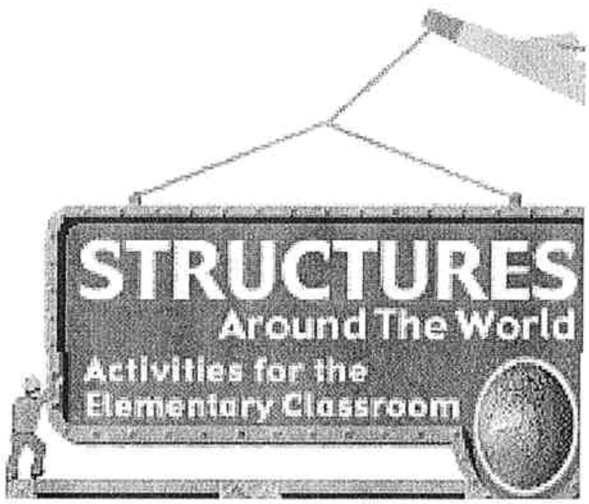


But if you make a toothpick triangle, the situation changes. The only way to change the angles of the triangle is by shortening one of the sides. So to make the triangle collapse you would have to push hard enough to break one of the toothpicks.

Mathematics Publications

Scale and Structures

The Exploratorium has published a number of books that have a mathematics orientation. *Scale and Structures,* available through the Exploratoriumweb site, www.exploratorium.edu, introduces elementary students and teachers to a number of measurement and geometry concepts. The image below is modified from the Exploratorium web pages.



Straws and Pins
Garden Poles

Cylinders and Scale
Clay Beams and Columns
Skewers and Garden Poles

Clay Bridges
Paper Bridges
Newspaper Bridges

Math Explorer

The newest Exploratorium mathematics publication is *Math Explorer*. Recognizing that middle school mathematics was a failing point for U.S. teenagers, the staff undertook a project to publish math activities intended for use with out-of-school youth development programs. NSF funded development of this project, and it is currently in the trial-testing phase.

Fifteen youth development programs, in California and other states, are testing 12 different activities each, from a total of 35, and providing feedback to staff about their usefulness and effectiveness with children. The programs also provide information about activities that they do not use and give the reasons why. Although most activities have met with approval, the staff has discovered that issues like reading level affect success. In a slightly modified format, *Math Explorer* is being tested by families for home use.

Lori Lambertson, the Exploratorium's project activity developer, has built-in components to make the book useful and friendly. Activities are charted with the corresponding California and

NCTM Standards and organized by skills or practices introduced. This latter component may prove most attractive to the community-based groups and families.

Interestingly, many potential consumers of mathematics are looking to remediate students on what they perceive to be the core mathematics skills. Although mathematics enthusiasts may find this step dismaying, the Exploratorium is creating a bridge by designing activities that excite learners and "sneak" in the skills practice. Additional information about *Math Explorer* can be found at http://www.exploratorium.edu/math-explorer.

Mathematics Exhibits

Many Exploratorium floor exhibits engage visitors in exploring light, sound, and perception. In the center of the hall, on the mezzanine, are a number of exhibits intentionally designed for mathematics exploration. This is a relatively small area in comparison with the overall layout. It was developed in the early 1980s, with funds from the Ford Foundation.

Of course, mathematics can be found in other Exploratorium exhibits as well. The challenge is in helping the visitor to know what mathematics is being presented. This is a significant question that Exploratorium staff are pondering. The mathematics is obvious to visitors who have a strong conceptual understanding of the discipline. However, most Exploratorium visitors are attracted by the phenomena and the investigative play that exhibits offer. Often they are unmindful of the underlying mathematics. To help bring out the mathematics content, the Exploratorium is considering a variety of approaches, including the use of hand-held computer technology.

Table 4 names and describes some of the exhibits that address mathematics. As you will see, most of the mathematics in the exhibits appears to be appropriate for middle school and high school levels.

Table 4. **Mathematics Exhibits at the Exploratorium**

Exhibit	Description	NCTM Standards	Grade Level
Gravity's Rainbow	Several balls, spaced unevenly, roll down an adjustable ramp; the visitor tries to land them in evenly spaced cups.	Standard 2 – Patterns, Functions, and Algebra (items 2.1 and 2.3) Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.4) Standard 5 – Data Analysis and Statistics (items 5.1 and 5.3) Standard 6 – Problem Solving (items 6.3 and 6.4) Standard 9 – Connections (item 9.3)	7–11
Make Bubbles	Create soap bubbles of various shapes.	• Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3,. and 3.4) • Standard 9 – Connections (items 9.1 and 9.3)	10–12
Radioactive Decay Model	Use cubes to represent the decay of elements over time.	• Standard 2 Patterns, Functions, and Algebra (items 2.1, 2.2, and 2.3) • Standard 5 – Data Analysis Statistics, and Probability (items 5.1, 5.2, and 5.3) • Standard 9 – Connections (items 9.1 and 9.3)	9–12
Lens Table	On an optical bench, visitors construct lens arrangements that enlarge or reduce an image.	• Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.3) • Standard 5 – Data Analysis Statistics, and Probability (items 5.1) • Standard 6 – Problem Solving (item 6.1) • Standard 9 – Connections (items 9.1, 9.2, and 9.3)	10–12

Exhibit	Description	Standard	Grade Level
Big Wheel	Making various patterns on a revolving cylinder creates music. Different patterns play different notes.	• Standard 2 – Patterns, Functions, and Algebra (item 2.1) • Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3) • Standard 6 – Problem Solving (item 6.1) • Standard 9 – Connections (items 9.1 and 9.3)	K–12
Big Chair, Little Chair	This exhibit allows the visitor to develop an understanding of what it means to double dimensions. All ages can study how much bigger the double-sized chair appears to be. Older children and adults can explore what happens to the surface area of the chair when the dimensions are doubled. Likewise, they can explore what happens to the volume of the seating space when dimensions are doubled. A guided discussion can help visitors explore the difference between doubling only one dimension of a *square,* versus doubling both dimensions. Also what happens when doubling only one or two dimensions of a *cube,* versus doubling all three dimensions.	• Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.4) • Standard 4 – Measurement (item 4.2)	6–9
Singing Coach	Try to match the pitch.	• Standard 2 – Patterns, Functions, and Algebra (item 2.1) • Standard 9 – Connections (items 9.1 and 9.3)	7–10

Partnerships

To achieve success with its various programs for mathematics teachers, the Exploratorium has developed relationships with local school districts and two nearby universities. The science center has an informal relationship with the University of San Francisco to support preservice teachers in learning. A more formal relationship exists with San Francisco State University, which allows credit to be given for workshops. The Exploratorium also has a program called Teachers in Residence, whereby teachers can spend a year working at the Exploratorium. It is this program that first brought mathematics educators into the science center.

Evaluation

Like most science centers working with mathematics, the Exploratorium has done almost no research on the effectiveness of programs or products. There are anecdotes—and the *Snacks* web pages permit teachers to chat with one another. There is data collection and feedback about how consumers use and respond to products, such as the trial testing of the *Math Explorer* book. But systematic collection of data, of the kind that would satisfy certain funders, is missing.

Lessons Learned

The "tyranny of the blackboard" can prevent mathematics teachers from using a hands-on approach.

According to Exploratorium staff:

> *[In teaching mathematics] there exists a strong tradition of using the blackboard, writing equations...and drawing graphs.*

For many teachers of mathematics, this tradition is hard to change. Some of the Teacher Institute participants were so wedded to the blackboard tradition that they felt they could never teach in the manner exemplified by the Exploratorium staff. For this group of teachers, stepping away from getting exact answers was too difficult. They want to find the "right" answer.

Unfortunately, real-world numbers obtained through investigations of the natural world (and with science center exhibits) may not always "fit." Sometimes data that are "good enough" have to be accepted. The process of trying to understand the stuff of the real world rather than "knowing the answers" can be disconcerting if one needs or expects precision.

For other teachers, mathematics concepts gained by studying phenomena were "cool." These teachers were able to see mathematics as a language to represent or convey natural, noticeable patterns in the world. They are the ones who clamor for seats at the half-day Saturday workshops the Exploratorium offers during the academic year.

The Saturday workshop experiences prepare the teachers in important ways. For example, in addition to gaining deeper understanding of mathematics concepts, they learn the kinds of challenges their own students have when trying to solve problems. Anticipating problems reduces student frustration and increases learning when experiments or activities are replicated in the classroom.

Shifting teacher learning experiences to more engaging and authentic use of mathematics may help defeat the notion that "teachers teach as they are taught." However, the tenacity of the image[14]—and reality—of teachers standing in front of the classroom, reviewing homework and showing how to use algorithms, is powerful (*Eighth-Grade Mathematics Lessons: United States, Japan, and Germany*, 1997). Certainly, it will take more than an Exploratorium "*Where's the Math?*" Teacher Institute to make changes in teacher behavior, but the science center's approach offers an alternative image for teachers to consider.

Many exhibits have "hidden mathematics," which need some form of interpretation for the visitor.

For many of the exhibits within the Exploratorium (and other science centers) there is an underpinning of mathematics. But the mathematics implicit in the phenomenon is usually not noticeable to the casual visitor without interpretation. Sometimes the visitor lacks the conceptual knowledge of the mathematics, but often the exhibit phenomenon simply doesn't elicit the mathematics the visitor *does* know. For the most part, visitors do not see through the lens of mathematics in viewing exhibits and consequently miss the connection.

This conundrum stimulated an ongoing question for the site-visit teams and staff at all of the science centers visited: Does mathematics reside in the exhibit or in the person? The perspective one takes on this question influences decisions such as determining copy on labels and deciding whether staff should "interpret" the mathematics or whether there are alternative strategies, perhaps untried, for elevating mathematics awareness.

The Exploratorium is considering a strategy of using hand-held computer devices that would enable the visitor to "play" with exhibits in novel ways. The Palm, or pocket, computer might also be programmed to include more extensive and detailed information about the exhibit.

Staff time for personal reflection and experimentation is an effective R & D strategy.

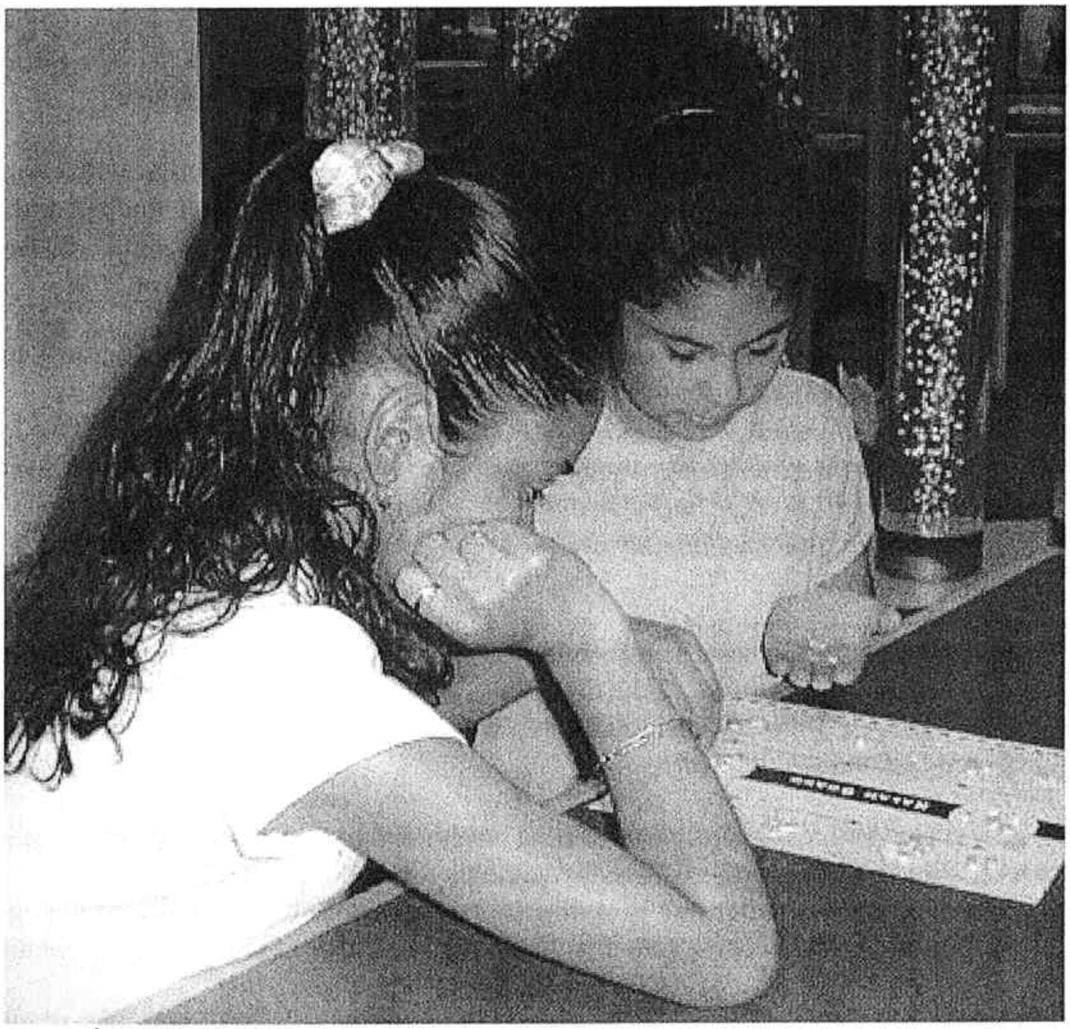
The Exploratorium recognizes that to do and present inquiry successfully, staff needs time to think. According to staff members, time for reflection and personal experimentation with exhibits, activities, and ideas is part of the Exploratorium's research and development strategy. The site-visit team was impressed with the freedom, opportunity, and encouragement given staff to try new approaches and to address new areas. We wondered if there was a correlation between the R & D approach and the number of grant-funded projects that the Exploratorium receives.

[14] This image mirrors the teaching efforts of the U.S. 8th grade mathematics teachers captured on the TIMSS video.

Investment in human resources reduces staff turnover and increases the institutional capacity to develop and deliver programs and exhibitions.

The site-visit team noted that both of the Bay Area science centers reviewed (the Exploratorium and the Lawrence Hall of Science) had high-caliber staff who earned significant salaries and had long careers with the science center. For the site-visit team, and indeed during later site visits, the discussions centered on the implications of constantly needing to train new people as a consequence of high turnover. Investment in human resources seems to result in low staff turnover and appears to reap the rewards of increased capacity to develop fundable projects, cultivate sustainable partnerships and secure grants.

To do mathematics in a science center, the institution must have access to mathematicians and/or mathematics educators. These can be individuals on an advisory board or part-timers working on special projects. But sustaining successful mathematics in an institution where science is king means sustaining a person on staff who is enthusiastic about mathematics content. High staff turnover threatens the potential for success.



Fort Worth Museum of Science and History
www.fortworthmuseum.org

Fort Worth Museum of Science and History

Extraordinary Learning Environments for Mathematics

The Fort Worth Museum of Science and History, in Fort Worth, Texas, is familiar with change and is following new directions. Started as a childrens' museum in the 1940s, it had become a science and history museum by 1960. In the mid 1990s, Fort Worth determined that it needed to think differently about its role in the community. According to Don Otto, president of the museum,

> *It was time to stand back and take the long view—for what will be in the next 50 years. Fort Worth has a collection, but is not a research institution. Where we had considered ourselves a teaching institution for half a century, we saw a strategic advantage in approaching our next 50 years as a learning organization.*

Staff took a hard look at their business. They brought in stakeholders from the community and thought about their values. They looked at the nature and quality of existing exhibits, and decided to try thinking in new directions. Ultimately, they chose a new guiding principle: Focus on learning, and you can change the world.

Fort Worth's goal is to create "extraordinary learning environments." The strategy is to establish successful partnerships and take small risks as investments for future teacher and family involvement. One tactic has been to invest in staff professional development and to encourage knowledge about inquiry, understanding about leadership, and skills in facilitating adult learning.

A big first step involved increasing Fort Worth's capacity to do inquiry. Staff members attended ASTC's NSF-funded Inquiry Institute for Teacher Educators and visited other science centers with inquiry discovery labs. Partnering with the Exploratorium has helped to cement this approach. Fort Worth also acquired a model for leadership development from the National Academy for Science Education Leadership, and staff made significant strides in building relationships with key stakeholders in the science education reform effort in Texas.

Forth Worth enhanced its facilities by creating a new Center for Science Learning, where teachers can meet and parents can gather. It created the Hands-On Science Learning Laboratory as a place for inquiry investigations. And it is in the midst of a fund-raising campaign to remodel and expand the museum itself.

Commitment to Building Relationships

The decision to focus on learning has had a powerful influence on programming and fund-raising at Fort Worth. The key has been to create relationships and build long-term partnerships with schools and to connect with other organizations trying to engage in systemic change in education. This philosophical stance brought Fort Worth into alignment with the Texas State Systemic Initiative (SSI)[15] and helped initiate the statewide Informal Science Network, in which Fort Worth has a key leadership role.

These connections prompted the Texas Science Teachers Association to establish a position on the board for informal learning institutions. Through Fort Worth's partnerships with universities and school districts, the museum participated in family learning events in mathematics and science, and staff member Colleen Blair began exploring the potential of working in preservice education.

The connections, collaborations, and relationships continue to build, with Fort Worth bringing in people from across the country to help them think about new directions. The strategy is to invest in audience development over the long term. Sometimes new audiences—like the preservice teacher education programs described below—are invited in at no charge. Participants are given passes for individual return visits. Ultimately, these visitors develop a new appreciation for what the museum offers and sustain the relationship through memberships and paid admissions.

Learning Is a Verb

Learning Is a Verb (Reynolds, 2000), a book written by one of Fort Worth's partners at Texas Christian University (TCU), provides a scholarly background for the development of the new philosophical approach of the museum. Staff members reference their new ideas against the notion of good learning experiences described in the book.

For example, like many science centers, the museum has science demonstrations for visitors. However, Fort Worth has consciously moved from delivering a scripted performance to having conversations with observers about the phenomena. Staff are intentionally exploring the concept of "social learning" as it applies to the museum visit, hoping to discover from the interaction better ways to insure learning. According to one staff member,

> *when learners engage in conversation about a phenomenon, they wrap language around their learning—they construct the concept and distill it to language in order to make it useful.*

Fort Worth invests in staff learning by bringing outside talent in to conduct professional development. Subsequent conversations among staff members continue the learning. Through this process the staff has adopted a slogan: "Focus on learning—it will change the world." They

[15] State Systemic Initiatives were funded by NSF during the early 1990s as a large-scale, statewide reform strategy for mathematics and science.

note that this focus has changed the institution, and it has changed how the staff design and implement programs. Ultimately, they believe the focus on learning will translate to high-quality experiences for visitors.

Learning and relationships are the foundation for the mathematics programs presented at the museum. This case study will focus on the preservice teacher workshops conducted in partnership with Texas Christian University. The museum also has a preschool program and kits for classroom use that have mathematical connections. The exhibit floor is being reconceived to integrate science and mathematics and to reflect what staff has learned about learning.

Mathematics at the Museum

Teacher Education

Preservice Teacher Education

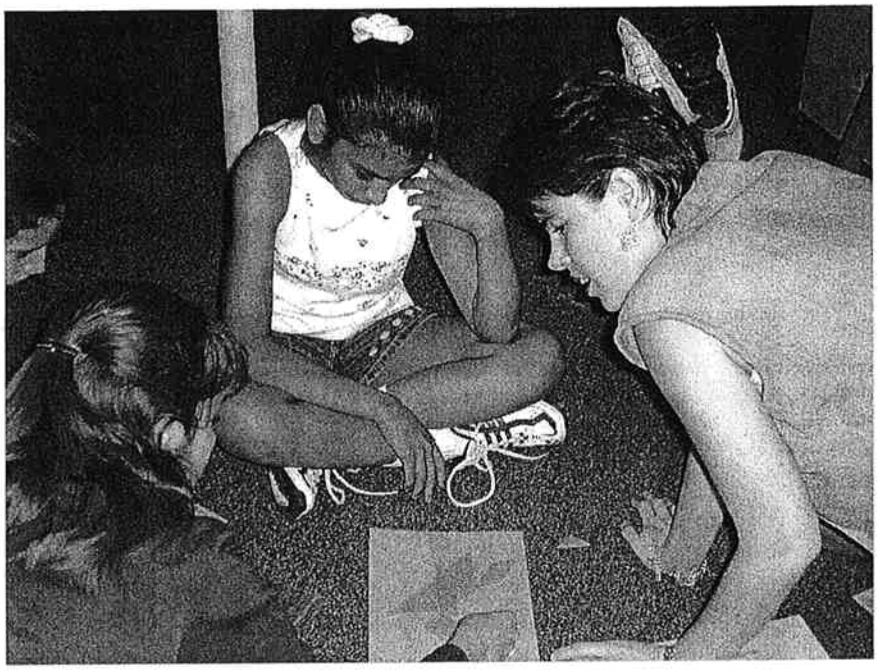
The *Preservice Teacher Education* program tells the story of museum transformation. As part of its transformation, Fort Worth intensified its relationships with local universities. In a pilot collaboration with Texas Christian University, the museum and the two TCU faculty members, Kathleen Martin and Sherry Reynolds, developed a program in mathematics education for college students enrolled in the teacher-certification program.

Mathematics educator Martin acquired a National Science Foundation grant to study inquiry and learning. She proposed bringing preservice teachers to the museum to work with visiting classes. Martin and Reynolds prepared the student teachers, while the museum arranged with the partner schools to bring in a class of students. The student teachers worked with small groups of children in a museum-organized lesson. The children were learning mathematics. The preservice teachers were learning about teaching.

The initial project began in 1993, and it has continued and expanded to include the University of Texas, Arlington. NSF no longer funds the core program, but the museum sustains it as an investment toward future business, using fees from the participating districts and other funds.

The museum invites local partner schools to bring students to the museum. These schools have a special relationship with the museum. The annual fee they pay guarantees that the partner schools' students have unlimited access to the museum. The museum also provides the schools with special opportunities for classes, such as those in which the student teachers work with small groups.

The university faculty members require their student teachers to go to the museum for preservice mathematics (and science) methods classes. Sometimes the student teachers work with the children on Piaget-type tasks. Most times the student teachers and children engage in what faculty members call "playful mathematics." The student teachers spend a significant amount of time teaching children and an equal amount of time reflecting on practice.



Grandfather Tang's story sets *Tangrams* into a multicultural context for mathematics and links math to literature.

The activity addresses the following NCTM Standards:
- Standard 3 – Geometry and Spatial Sense (items 3.3 and 3.4)
- Standard 6 – Problem Solving (items 6.1, 6.3, and 6.4)

The student teachers meet with the children in the Center for Science Learning, where each student teacher is assigned to work with a small group. Class begins when the Fort Worth staff person introduces the program and launches the activities for the day.

On the day of our site visit, the students were engaged in mathematics games: Tangrams, Mancala, and Magic Squares. The children rotated through the various activities, working with the student teachers. The task for the preservice teachers was, in part, to help the students understand the game and figure out the problem-solving strategies. At the same time, these fledgling teachers were observing how the students were learning the mathematics.

At the conclusion of the lesson, the children were brought together as a group to summarize their findings and insights. Anne Herndon, from the Fort Worth staff, conducted the lesson, making sure the underlying mathematics concepts were explored through the games. At the end, she dismissed the children and gave the podium to the TCU faculty. During the ensuing discussion, the preservice teachers described what they noticed and considered what it meant for them as future teachers in a school classroom.

At first, the novice teachers shared how their students figured out the problems they were given. They had been instructed to pay careful attention to how children responded to the games, to the challenge of finding solutions and winning. Shortly the conversation evolved to a discussion of mathematics.

While it was not clear from the discourse whether the teachers had a strong conceptual understanding of the mathematics, they recognized how powerful this way of mathematics learning is. One of the teachers said,

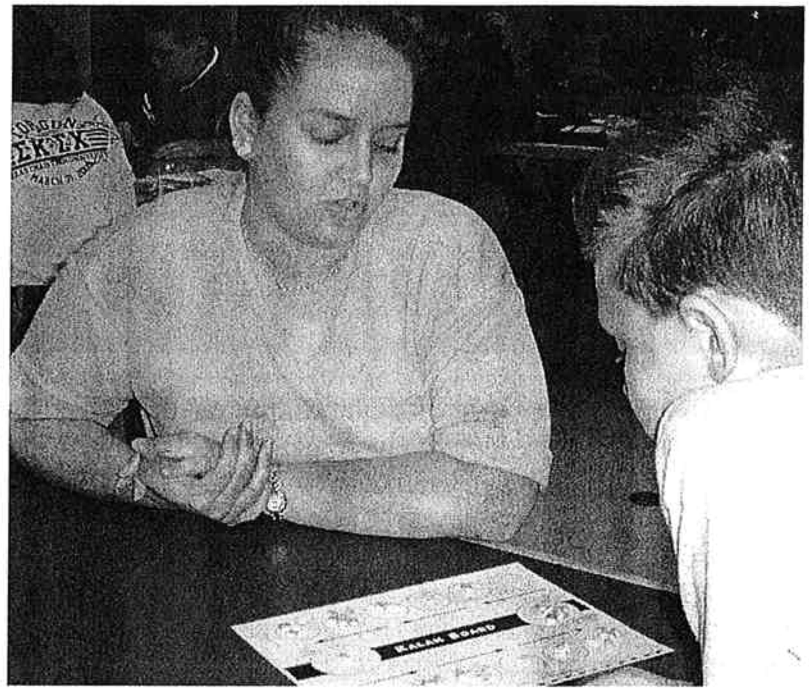
> *If I had been taught this way I would not have wasted my time avoiding school. This kind of learning would be fun and I would know how to do the mathematics.*

There was consensus among the student teachers that they would teach this way once they had their own classroom. They discovered that the children later tried the same activities on their own and also found the youngsters engaged in other learning activities at the museum. The student teachers were thoughtful about using similar activities and materials in the classroom, because they could observe the way children responded. The student teachers were very positive about the whole experience and felt encouraged to think about teaching in new ways.

This preservice teacher found the boy using skillful strategies to defeat her in *Mancala*[16]. This strategy game develops accuracy in counting and the ability to plan ahead and solve problems. It is appropriate for a range of ages, from the very young (age 5 or 6) to adults. Discussions between student teachers and the children help both parties refine their mathematical thinking and learn to communicate.

The lesson help students understand these NCTM Standards:

- Standard 1 – Numbers and Operations (item 1.1)
- Standard 2 – Patterns, Functions, and Algebra (item 2.1)
- Standard 5 – Data Analysis, Statistics and Probability (item 5.1)
- Standard 6 – Problem Solving (item 6.3)
- Standard 8 – Communication (items 8.2 and 8.3)

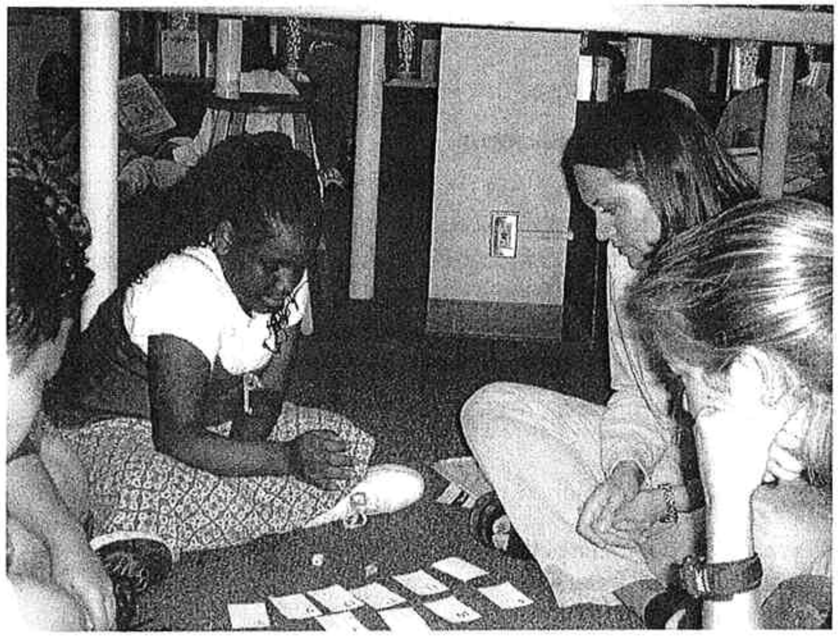


[16] Called Mancala here, this game is also known as Kalah, the name used by the Lawrence Hall of Science.

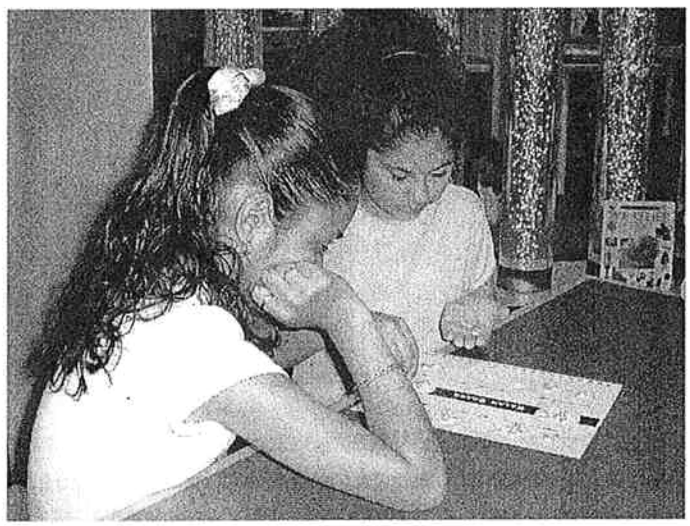
Magic Squares is a problem-solving activity that engages the learner completely.

This activity addresses

- Standard 1 – Numbers and Operations (items 1.1, 1.2, and 1.3)
- Standard 2 – Patterns, Functions and Algebra (item 2.1)
- Standard 6 – Problem Solving (items 6.1 and 6.3)



The photograph at right shows two of the children, who had participated in the preservice class at Fort Worth, playing one of the games they learned during the session.



Several of the students who had participated in the workshop with the student teachers set out to explore more mathematics. They decided to figure out what it would take to build a pyramid of glasses.

Exhibits and Public Programs

Kits, Carts, and Collections

Fort Worth has always had a teaching collection, which is used in its educational programs, as well as a permanent collection utilized for exhibitions on the museum floor. Now the museum is pulling items from the teaching collection, putting them into kits, and sharing them with teachers. Although for the most part there is no curriculum included, teachers (and home schoolers) can use the kits. Volunteers also use kit materials on "theme" carts, where they present objects, ideas, and interpretations for visitors.

As can be seen from this kit on the floor, Fort Worth is using authentic objects from its collections to enhance learning.

All kits and carts at Fort Worth are currently undergoing review in light of the museum's new direction. At the time of the site visit, one hands-on program, *Multicultural Math,* had been discontinued, and during discussion staff presented it as a negative exemplar for the path the museum seeks. Staff felt that it neither constituted an "extraordinary learning experience," nor linked with the NCTM Standards. One staff member articulated the essence of what they were seeking in a mathematics experience and explained the failings of the *Multicultural Math* kit:

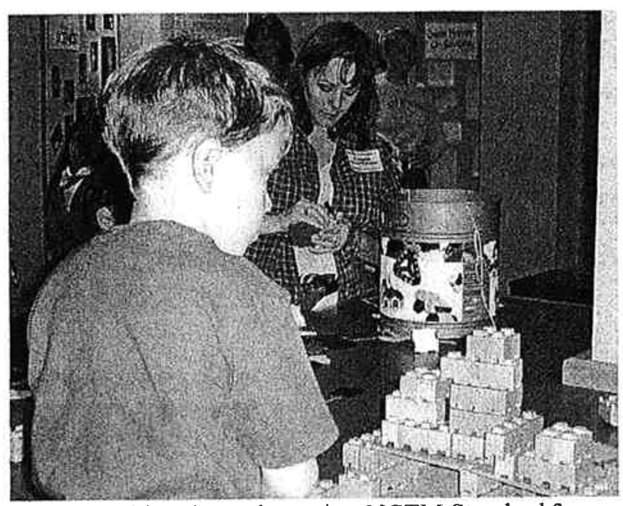
> *There is playfulness that comes from mathematics, but it comes from the experience. If the mathematics language comes from it naturally, as a useful tool to understand the experience, that is the best. That is the touchstone.*
>
> *If the experience is real, then the mathematics should be the real stuff coming out of the experience. The frustration for learners within schools is that the problems are contrived and the mathematics is tedious. If the activity is too transparent in generating a mathematics skill, then it shows and it is boring.*

Fort Worth staff members want visitors to think about mathematics more broadly—as more than just doing arithmetic. The museum has struggled with what is means to do mathematics successfully. They continue to experiment with ways of providing experiences that lead to deeper understanding of mathematics principles. Once again Reynolds helps guide their thinking:

> *Because of the nature of mathematics, which is relational, all mathematics is in the person, not the exhibit. The mathematics isn't in the exhibit; it is in the relationship with the phenomena in the mind of the viewer.*
>
> *The person constructs all knowledge. The environment [exhibit] can trigger the construction of ideas. You can't assume it will deliver information, but the best exhibits disturb you about the ideas you hold. The question is, At what point is the disturbance solid enough that you can say you have done mathematics?*

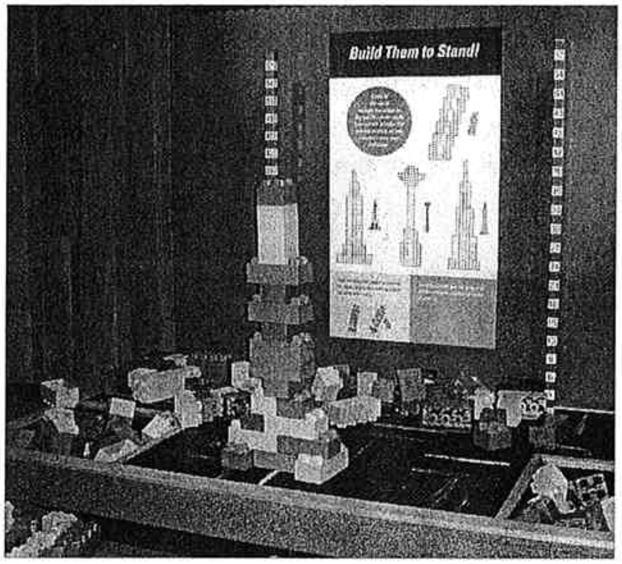
Exhibits with Mathematics

The exhibit designers at Fort Worth now acknowledge that visitors do not necessarily learn what a designer intends. People take away different things from an exhibit, based on what they started with and what they were seeking to know. Exhibits give an experience; the meaning-making depends on the individual. However, provocative exhibits and experiences that cause conceptual disequilibrium, may encourage individual reflection and additional investigation and ultimately trigger understanding.



The two Fort Worth exhibits shown invite the young visitor to explore ideas in mathematics. NCTM Standard 3 – Geometry and Spatial Sense, Standard 6 – Problem Solving, and Standard 10 – Representation are explored through computer or real-object exhibits.

The transition from displaying static exhibits to designing learning experiences has taken time. Collections are being used as hands-on objects for learning, and floor interpreters engage visitors in asking and answering questions. Each floor exhibit is observed for the kind of visitor interaction that suggests a deeper engagement with the ideas presented. The staff is examining the exhibits to see that they meet the criteria of extraordinary learning environments and will drop or adapt those that do not fit the criteria.

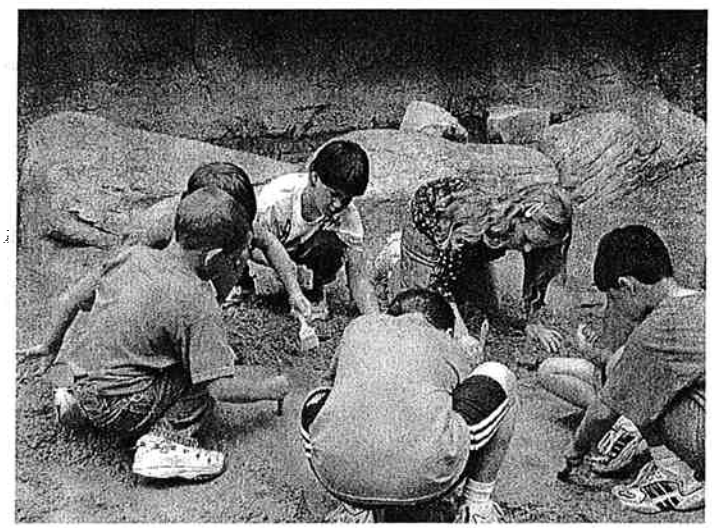


Built Them To Stand is an exhibit that invites visitors to build towers. It addresses these NCTM Standards:

- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 6 – Problem Solving (item 6.1 and 6.4)

Dinosaur Dig has children looking for bones. With an intentional mathematics extension to the activity, this exhibit could easily address these NCTM Standards:

- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.2, and 3.4)
- Standard 4 – Measurement (item 4.1)
- Standard 5 – Data Analysis (items 5.1 and 5.3)
- Standard 9 – Connections (items 9.1 and 9.3)
- Standard 10 – Representation (item 10.1 and10.3)



Preschool Programs

Fort Worth is widely recognized for its preschool programs. Parents bring children to the preschool early in the day. During their time at the museum, the children are in classrooms filled with lots of free-choice activities.

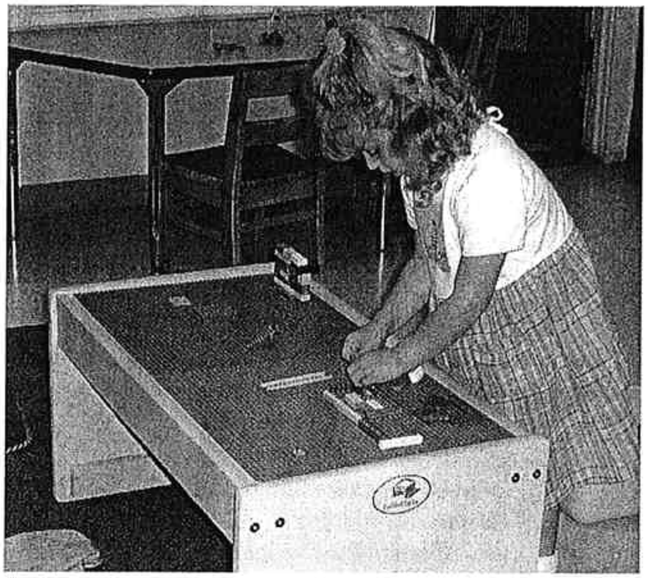
In the photo at left, children move about the space, doing what they wish. But the staff has organized the space and set out enticing activities that enable children to have high-quality early experiences with both mathematics and science.

Counting out objects, distributing equal numbers for each child is an early learning activity. It addresses

- Standard 1 – Numbers and Operations (items 1.1 and 1.2)
- Standard 4 – Measurement (items 4.1 and 4.2)

Museum staff remain available to help children when necessary. At this station, children learn about making sound. Changing the rubber band's length changes the pitch. This builds understanding of

- Standard 2 – Patterns, Functions and Algebra (items 2.1 and 2.3)
- Standard 4 – Measurement (Items 4.1 and 4.2)
- Standard 9 – Connections (items 9.1 and 9.3)



A young girl begins the mathematics learning experience by playing with Legos.

The NCTM Standards connected here are

- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.2, 3.3, and 3.4)
- Standard 4 – Measurement (item 4.1)
- Standard 6 – Problem Solving (items 6.1, 6.2, and 6.3)
- Standard 10 – Representation (item10.2)

Children use scoops and measuring cups to pour sand into larger containers. The preschool experience with sand and containers helps young children understand the idea of capacity and conservation of volume and develop such understandings as it is not just the height of a container that determines how much it can hold.

NCTM Standards observed are

- Standard 1 – Numbers and Operations (items 1.1 and 1.2)
- Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.4)
- Standard 4 – Measurement (item 4.1)
- Standard 8 – Communication (items 8.1, 8.2, and 8.3)
- Standard 9 – Connections (items 9.1 and 9.3)

Lessons Learned

Algebra is not the cause of mathematics failure; it is where lack of understanding becomes apparent.

One of the most important lessons uncovered during the Fort Worth visit had to do with understanding mathematics learning. During the visit, the team met with TCU professor Sherry Reynolds, who provided insight regarding child development, teaching and pedagogy, and mathematics learning. One of the questions posed to Reynolds had to do with apparent mathematics failure at the middle school level. Based on her research in psychology and her observations at Fort Worth, the professor explained:

> *We see children failing in algebra, but algebra is not the cause. Rather, algebra is the place where what you don't understand about mathematics and arithmetic becomes apparent.*

According to Reynolds' research, we can best improve understanding of algebra by improving children's understanding of arithmetic. She suggests that in the United States there is a specific problem with fractions, ratios, and proportions, but she notes that Swiss psychologist Jean Piaget said that an inability to do arithmetic is a uniquely American problem. Says Reynolds:

> *If otherwise perfectly bright children can learn other subjects, but not mathematics, then something is wrong with how we are doing it.*

She is very interested in the active approach toward learning mathematics inherent in museum-type interactions. According to Reynolds, science centers can make potentially powerful contributions toward elevating achievement and understanding of critical mathematics concepts.

Taking risks and implementing new visions will build a stronger future for the museum

Although the decision by Fort Worth to rethink its mission and direction caused staff to step back from traditional practices, they believe the risk will have enormous payoff. Establishing and holding to the standard of "extraordinary learning environments" will, in the long term, bring success. They believe it will stimulate deeper, longer relationships with partners and stakeholders and result in quality-of-life improvements for the community.

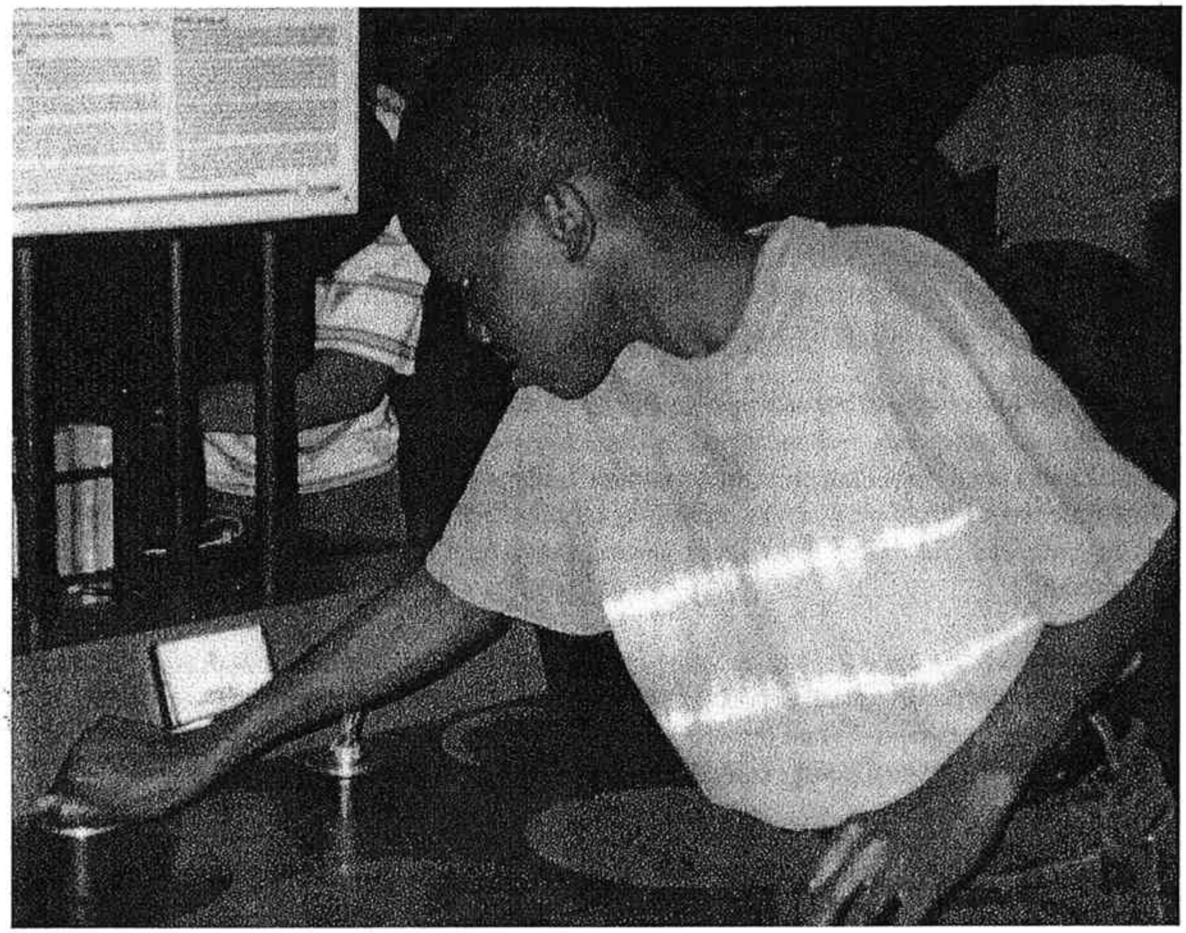
In the same way that notions of equity permeate Lawrence Hall and inquiry drives the Exploratorium, the focus on learning at Fort Worth is a thread of their corporate culture. Although the site-visit team viewed mathematics programs and products at the beginning of the process, it was clear changes had occurred. Conversations about exhibits and future plans reflected concern for deep and long-term learning, while programs were being modified to deepen the intellectual engagement of participants.

Systemic changes in the museum and in public education require new roles and new relationships among partners.

To change the system means that the role of each component and its relationship to other components must be rethought. Not only is Fort Worth going through a fundamental change internally; the museum is also seeking to change its role in the Texas education system. Stepping into a pivotal role as initiator of the Texas Informal Network, as part of the state's Systemic Initiative, resulted in a different view of what museums can contribute toward improved educational achievement. Moreover, this role altered how the museum viewed itself relative to the other informal "competitors." It stimulated the commitment to create partners and build relationships on a statewide level.

This view of cooperation rather than competition has resulted in much of the goal setting and role rethinking noted earlier. Fortunately, the museum leadership is willing to take the long view on systemic change.

In public education, systemic change addresses both mathematics and science achievement. Significantly, the museum is committed to continuing program and product creation in which mathematics and science are integrated. The network of relationships Fort Worth has built regionally and nationally will help the museum succeed.



Pacific Science Center

www.pacsci.org

Pacific Science Center

Mathfinder and the Museum

Pacific Science Center is located in the heart of Seattle, Washington. Like many science centers, it serves the local community, school groups, and tourists. With traveling blockbusters supplementing an array of permanent exhibits and two IMAX theaters hosting large-format films, the science center is attractive to locals and tourists alike.

Mathematics has had a long-term presence at Pacific Science Center. It is specified in the museum's mission statement, and mathematics educators serve on the education advisory committee. During the 1980s Pacific Science Center offered teacher workshops in mathematics, which were focused on problem-solving strategies and provided teachers with hands-on lessons and teaching materials. However, these were phased out as the NCTM Standards became more widely adopted.

Mathematics gained "new life" at Pacific Science Center when Washington adopted the Essential Academic Learning Requirements (EALRs)[17]. These K–12 educational standards are tied to state-level, high-stakes exams. When the first results in mathematics came in, the scores were abysmal, and Pacific Science Center seized the opportunity to design a new mathematics outreach program.

For this case study, the site-visit team focused primarily on the Science on Wheels program and its *Mathfinder* van. But the team also "unpacked" the mathematics in floor exhibits, using "a mathematics lens" to explore the different science disciplines presented in the science center exhibits.

Pacific Science Center's education programs serve more than 162,300 students, teachers, and other adults each year. What makes the education offerings notable is that Washington's Office of the Superintendent of Public Instruction (OSPI) provides significant funding for these programs through a line item in the state budget. Through its Science on Wheels program, Pacific Science Center demonstrated that it could serve schools beyond the metropolitan area of Seattle. The science center obtained visibility for its programs by notifying legislators whenever the van visited schools in their area. This strategy helped guarantee audience awareness and government support for what the science center could do for schools.

[17] The Essential Academic Learning Requirements (EALRs) were developed based on the NCTM Standards and are similar in structure and content. Use of the NCTM Standards for this publication permits more generalized use.

Mathematics at Pacific Science Center

Van-Program Structure

The Science on Wheels van programs are Pacific Science Center's key strategy for maintaining connections with school districts across the state. Now a high-visibility program, Science on Wheels got off to a modest start in 1973, when, with gasoline in short supply, school districts couldn't afford to visit the science center. Confronted by declining revenues, staff decided to bring the science center to the schools. They set up a pilot program with one local school district, filled a station wagon with hands-on science lessons and a few floor exhibits, and drove out to a nearby school.

From that initial visit, the program grew in scope and in service area. In the beginning, Science on Wheels staff taught a handful of science lessons in classrooms. Now the van program includes tabletop exhibits, a menu of lessons, and an all-school assembly. The program logs 540 van days each school year, and service is statewide[18]. There are six Science on Wheels programs: *Physics on Wheels, Space Odyssey, Teach Every Child How* (T.E.C.H.), *Blood & Guts* (human physiology), *Rock and Roll* (geology), and, newest of the offerings, *Mathfinder*[19].

The basic program structure varies only slightly among the different vans. Generally, there is a 30-minute all-school assembly, an exhibit hall of 20 to 30 tabletop exhibit devices, and a set of 6 to 9 classroom lessons. Programs require two or three Pacific Science Center staff and a number of school-based volunteers.

The school chooses in advance which van it wants and reserves a day or more for the program. Staff arrives an hour before the opening assembly to set up the exhibit hall and train volunteers. Volunteer support is crucial since the staff spends the day teaching lessons in the classrooms. The volunteers help set up the exhibits and learn to do simple maintenance. They are also trained to ask questions of the students and provide simple directions for exhibit use.

The all-school assembly serves as an introduction and stimulates excitement. Each program offers students some challenge to resolve during exhibit visits and in classroom lessons. Some assemblies are done in costume, and one includes teachers from the schools as performers alongside the staff.

Teachers preselect a 45-minute classroom lesson appropriate to their grade level from a menu of options. Class instruction is for a maximum of 32 students, a constraint imposed by the materials available per session. The classroom teacher stays to monitor the students and to observe the instructional strategies—a serendipitous professional development benefit. Each lesson is extended with a leave-behind sheet for the teachers and a take-home sheet for families and children to do together.

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[18] The van concept has been replicated in other science centers in the United States and internationally.
[19] In addition to the six OSPI van programs, there are two middle school van programs: Waste Busters, funded by King County and Brain Power, funded by the National Institute of Health.

The **Mathfinder** *Assembly*

The purpose of the *Mathfinder* assembly is to orient the students and staff to the structure of the day and prepare them with conceptually stimulating ideas in anticipation of the classroom and exhibit hall activities. The assembly explores the concept of probability by having students consider the likelihood of reaching a specific phone number (for example, that of the President of the United States) by dialing 10 random numbers, 0 to 9. Students then participate in a demonstration to illustrate the concept. Scale and perspective are introduced by showing the audience a trick photograph of dinosaurs, alive and well and living in Seattle. The program concludes by encouraging students to view mathematics as a powerful tool in daily life.

Mathfinder *Classroom Lessons*

The blueprint for designing a van program was well established before *Mathfinder* was created. Historically, the van programs had offered K–3 and 4–8 grade lessons. But for mathematics, the staff established an advisory group of teachers and asked them for recommendations. Following their advice, the team planned and developed three lessons each for grades K–2, 3–5, and 6–8.

To narrow choices down and to build the connection with schools, the development team started with the EALRs. A survey of teachers and administrators helped the team select which of the Learning Requirements needed the most support. Teachers suggested a variety of activities that required too-expensive materials or that were too difficult to do in a hands-on way. Ultimately, the developers chose to focus on five math topics: probability, statistics, geometry, logic and problem solving, and measurement.

Pacific Science Center staff developed the mathematics lessons and then did a pilot test. The first prototypes were tested in three sessions: with school groups, with families, and on the floor of the science center. Observations and feedback allowed the staff to modify products before the van made its debut. The table of *Mathfinder* lessons that follows denotes both the topics and the NCTM Standards addressed by each.

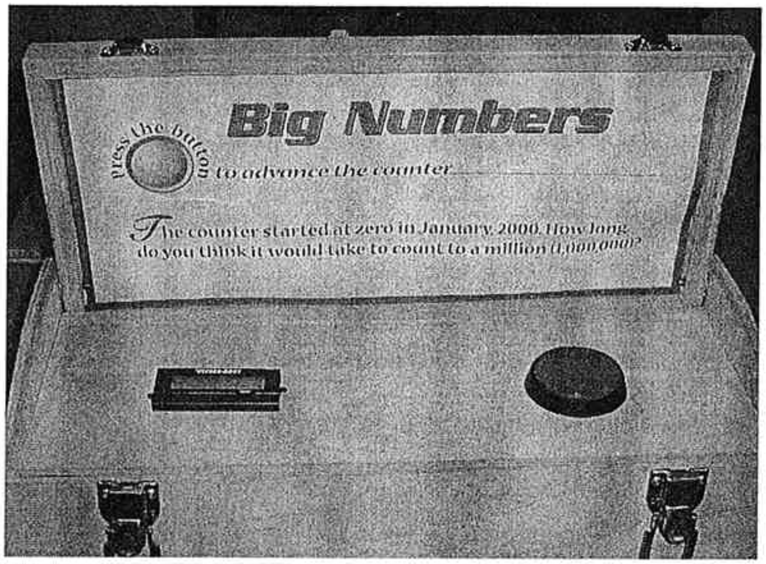
Table 5. Mathematics Lessons for Pacific Science Center's *Mathfinder* Van

Math Lesson	Topic	NCTM Standards	Grade Level
Sometimes Never	Probability	• Standard 1 – Numbers and Operations (item 1.3) • Standard 4 – Measurement (item 4.1) • Standard 5 – Data Analysis, Probability (items 5.4 and 5.1)	K–2
Mirror Image	Geometry	1. Standard 1 – Numbers and Operations (item 1.3) 2. Standard 2 – Patterns, Functions, and Algebra (item 2.2) 3. Standard 3 – Geometry and Spatial Sense (item 3.3) 4. Standard 5 – Data Analysis, Probability (item 5.1) 5.	K–2
Measure Up	Measurement	• Standard 1 – Numbers and Operations (item 1.1) • Standard 4 – Measurement (item 4.1) • Standard 2 – Patterns and Functions (item 2.2)	K–2
Pentominoes	Geometry	• Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3) • Standard 6 – Problem Solving (items 6.1 and 6.2) • Standard 7 Reasoning and Proof (item 7.2)	3–5
Collect All 4	Probability	• Standard 1 – Numbers and Operations (item 1.3) • Standard 5 – Data Analysis, Probability (item 5.3)	3–5
Gravity Central	Problem Solving	• Standard 6 – Problem Solving (items 6.1 and 6.2)	3–5
What Are the Odds?	Probability and Genetics	• Standard 5 – Data Analysis, Probability (items 5.3 and 5.4)	6–8
Fabulous Fractals	Geometry	• Standard 3 – Geometry and Spatial Sense (item 3.3) • Standard 6 – Problem Solving (items 6.1 and 6.2) • Standard 7 Reasoning and Proof (item 7.2)	6–8
Net Shape Navigator	Geometry (3-D)	1. Standard 3 – Geometry and Spatial Sense (item 3.4) 2. Standard 6 – Problem Solving (items 6.1 and 6.2) 3. Standard 7 Reasoning and Proof (item 7.2)	6–8

Tabletop Exhibits

It was a challenge for exhibit developers at a science center to think about mathematics exhibits. To push their thinking about appropriate mathematics exhibits, the Pacific Science Center design team reviewed children's books and brainstormed ideas for interesting exhibits. They also had to think about the EALRs and the developmental appropriateness of concepts. In the following photographs, some of the exhibits are presented, along with the corresponding NCTM Standards.

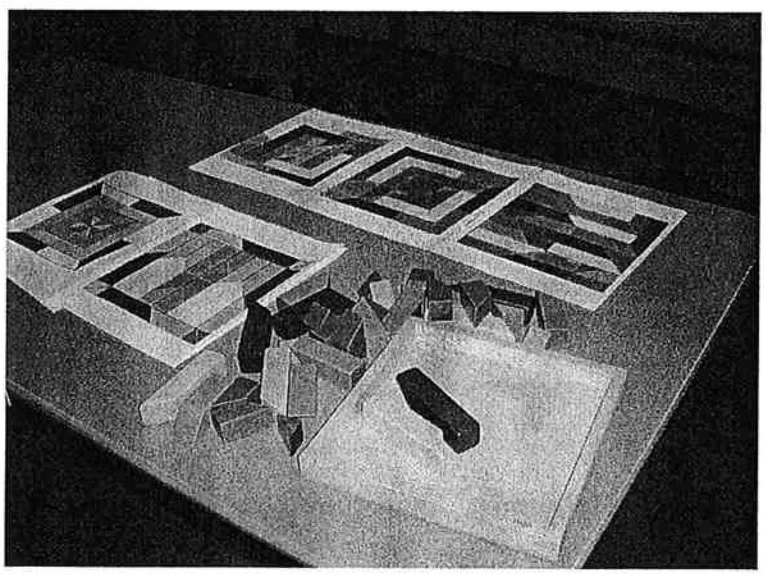
Standard 1 – Number and Operations



Big Numbers is a counting machine that began its count at zero in January 2000. Visitors are asked to press a button to advance the counter. The macine then asks the visitor to consider how long it will take to count to 1,000,000 (1 million). The exhibit addresses

- Standard 1 – Numbers and Operations (items 1.1 and 1.1)

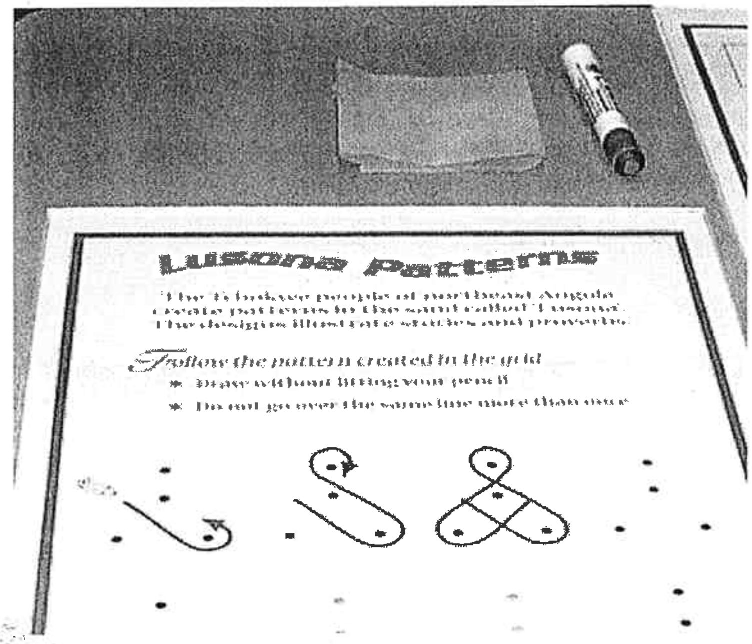
Standard 2 – Patterns, Functions, and Algebra



Pattern Block encourages visitors to replicate patterns using the colored blocks. It specifically addresses

- Standard 2 – Patterns and Functions (items 2.1 and 2.1)

Standard 3 – Geometry and Spatial Sense



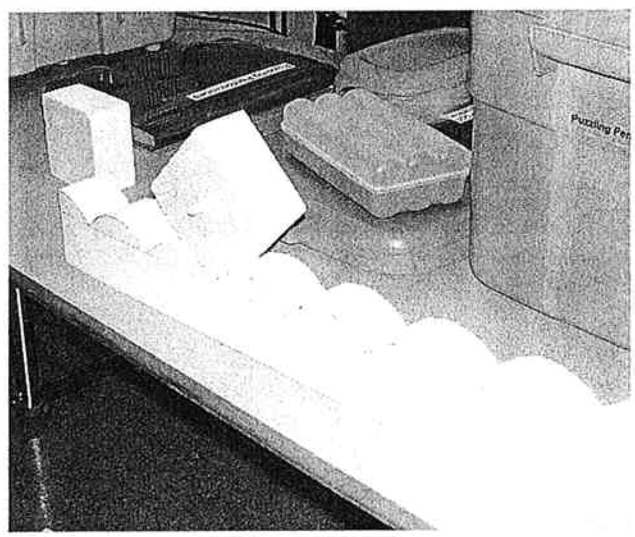
Some *Mathfinder* exhibits address more than one Standard. Here a traditional northeast Angolan activity, *Lusona Patterns,* invites visitors to draw patterns without lifting the pen from the paper and without crossing lines twice.

Lusona Patterns addresses

1. Standard 2 – Patterns, Functions, and Algebra (item 2.1).
2. Standard 3 – Geometry and Spatial Sense (item 3.2)

Square Wheel invites the learner to figure out how to make a square object roll smoothly. The mathematics enters in as visitors analyze characteristics and properties of two- and three-dimensional geometric objects and use different representational systems, including coordinate geometry and graph theory.

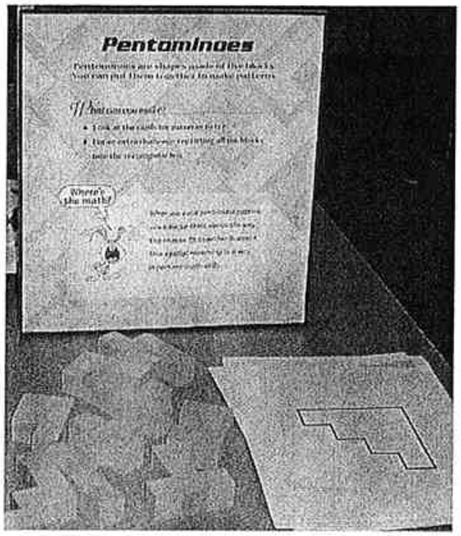
- Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3)

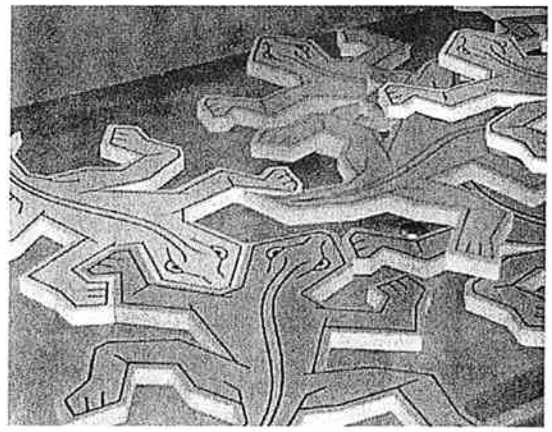


Other well-known science center exhibits or activities, such as *Pentominoes* (below, left) and *Escher Tessellation* (below, right) found their way into tabletop devices for use in schools.

The Pentomino blocks and their puzzles help to develop spatial reasoning. The class lessons provide the experience of creating the 12 Pentominoes, which requires work in spatial reasoning, and trial-and-error problem solving and can reinforce the difference between area and perimeter. It works well for students grade 3 and up. Those as young as 7 or 8 do well at creating the 12 pentominoes by working in groups or working with adults. Middle school and older students can tackle the harder puzzles and work to create their own set of blocks.

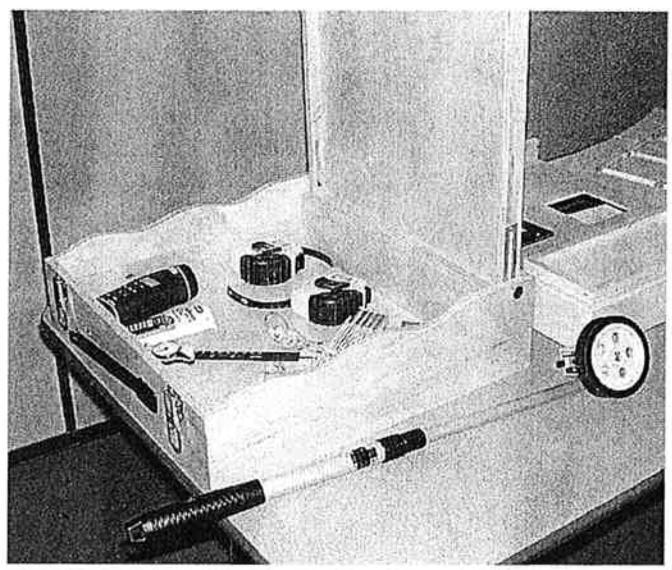
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3 and 3.4)
- Standard 4 – Measurement (item 4.1)
- Standard 6 – Problem Solving (items 6.1 and 6.3)





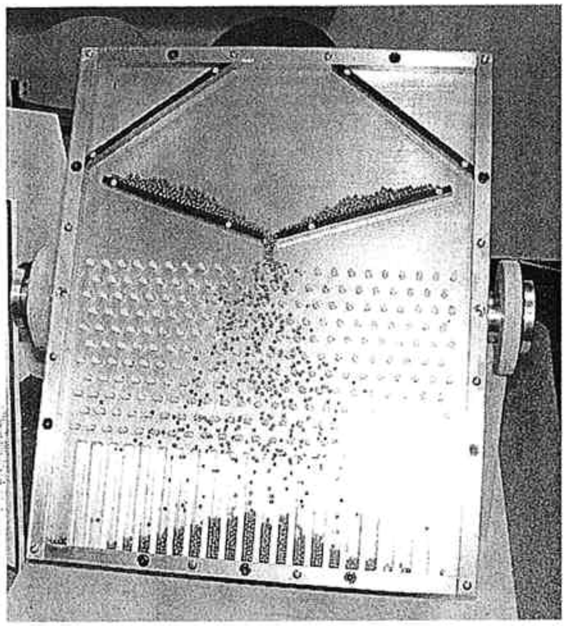
Standard 4 – Measurement

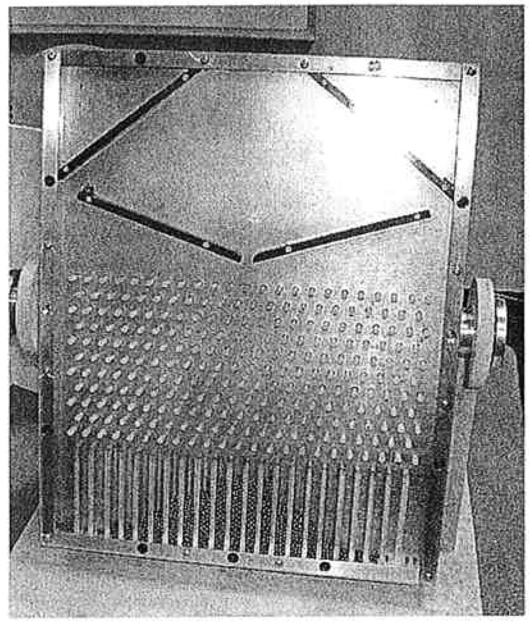
This box filled with tools for making measurements covers many of the Standard 4 – Measurement items.

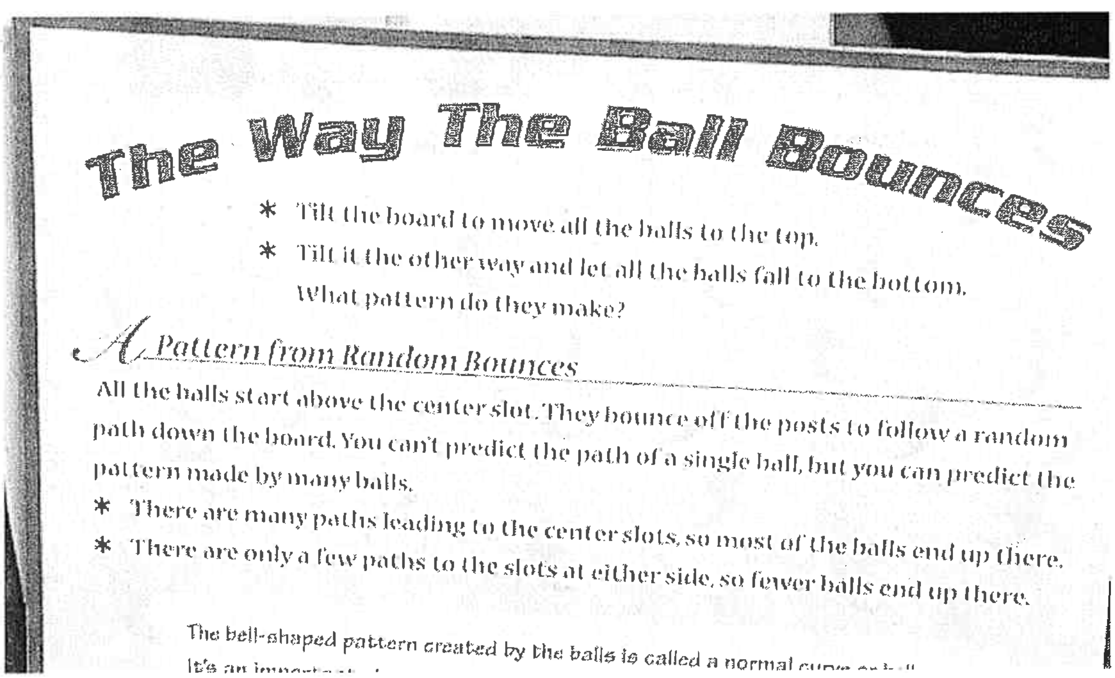


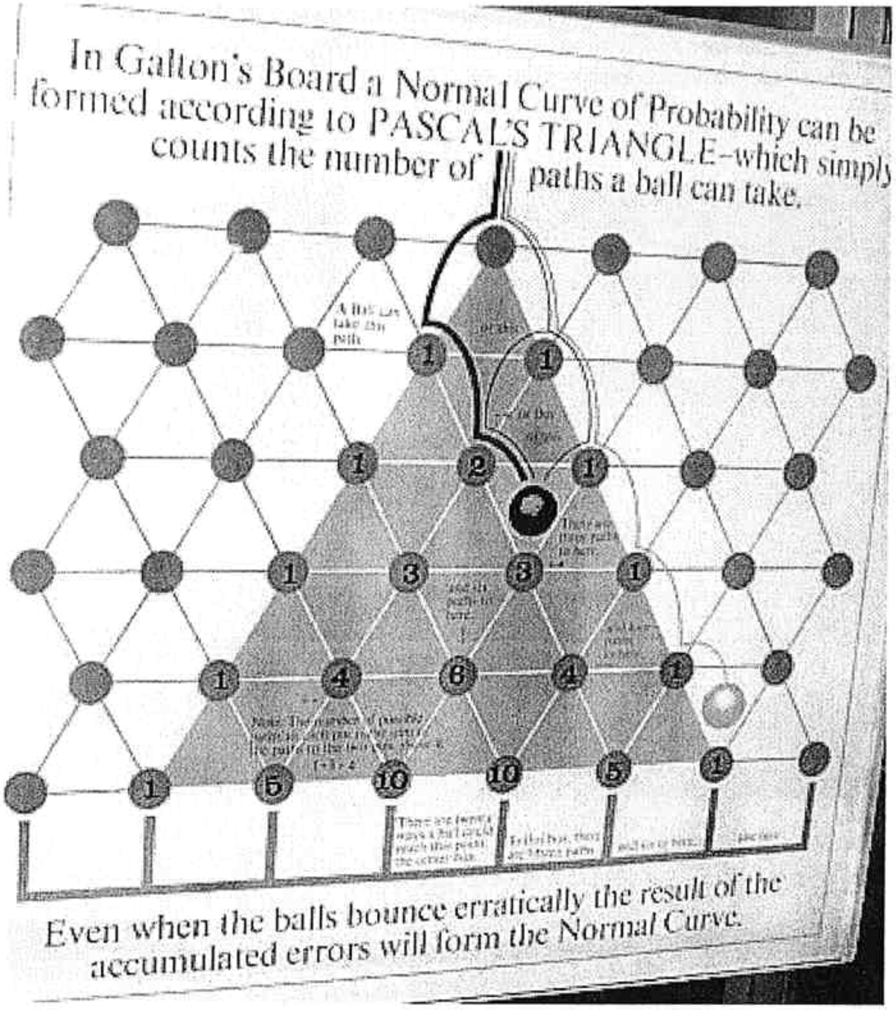
Standard 5 – Data Analysis, Statistics, and Probability

The normal-distribution exhibit is found in science centers in many guises. Pacific Science Center's *The Way the Ball Bounces* exhibit is miniaturized and made to travel. The following photographs show the exhibit in motion and the resulting distribution. This exhibit corresponds to NCTM Standard 5 – Data Analysis, Statistics and Probability (items 5.2.and 5.4).









Additional text from signage of "*The way the Ball Bounces*" exhibit

Funding

Costs for the Science on Wheels program include development expenses, materials and resupply costs, staff salaries, travel, and indirect costs. OSPI provides approximately two-thirds of the delivery costs of all the programs. The remainder comes from other sources, including Pacific Science Center's development office, which raises money to defray expenses and keep school fees as low as possible. Schools pay about 20 percent of the cost of a program, depending on how many schools the science center visits while traveling within a specific region.

The budget for the newest van, *Mathfinder*, was $350,000, split over two years. Two-thirds was spent in the first year to develop lessons, exhibits, and the assembly. The budget for lesson development was $69,000, exhibit development was $122,000; and the assembly development cost was $5,500. The van costs $27,500. The remaining funds will support first-year program delivery, but delivery costs exceed what is provided by OSPI.

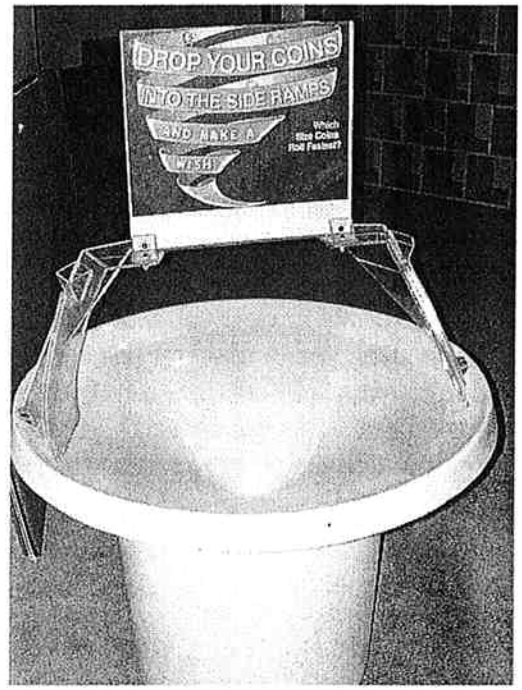
Evaluation

As at many science centers, full-scale evaluation is not a high priority at Pacific Science Center, and there are no systematic studies of how or what children are learning, nor any investigation of the ideas that teachers pick up during the visits. However, some effort has been made to find out what teachers and students have to say about the van programs. The staff asks classroom teachers to fill out questionnaires at the end of the lessons, offering comments about the age appropriateness of the lessons, the assembly or exhibits, and what students are saying. The contact person is also asked to fill out an overall evaluation form.

Mathematics Exhibits and Public Programs

Mathematics Found in the Science Exhibits

At the Pacific Science Center, many of the existing exhibits have mathematics connections that are not explicated directly. Mathematics is inherent in virtually every science discipline. The science is articulated in the copy, and made enjoyable in the exhibits. Mostly the mathematics seems obscure, but the connections could be made for the visitor. The following photographs illustrate this point, with the relevant NCTM Standards.



In Astro-Physics, the *Gravity Well* exhibit—found in many centers—invites visitors to drop coins or marbles into the well to observe the elliptical orbits that develop. It addresses

- Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.4).

Mirrors and prisms make connections between geometry and real-world phenomena. The photograph at left is the *Parabolic Mirror,* where visitors investigate parabolas and foci.

It addresses NCTM Standards

- Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3)

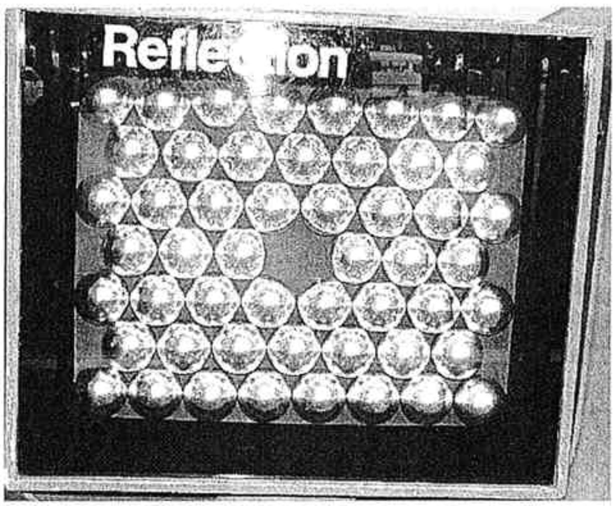
Below, at the left, is an exhibit on *Reflections*.

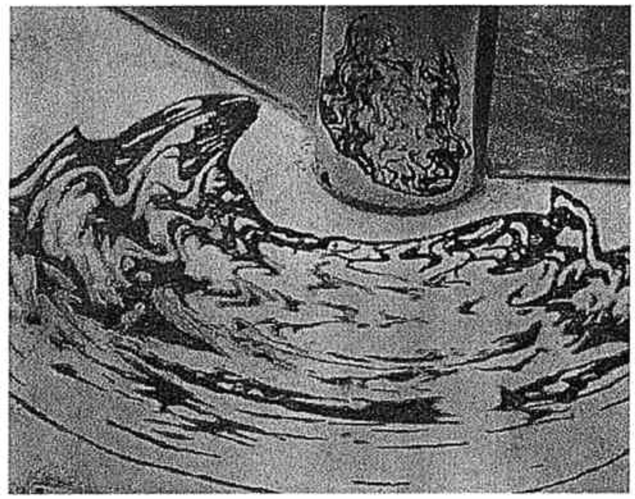
- Standard 3 – Geometry and Spatial Sense (item 3.1)

On the right is an example of Leonard da Vinci's *Anamorphic Art,* which relies on curved mirrors to make sense of the image.

The Standards addressed in *Anamorphic Art* are:

- Standard 3 – Geometry and Spatial Sense (items 3.2 and 3.4)
- Standard 5 – Data Analysis and Representation (item 5.1)
- Standard 6 – Problem Solving (item 6.3)
- Standard 8 – Communication (items 8.1, 8.2, 8.3, and 8.4)
- Standard 10 – Representation (items 10.1 and 10.3)

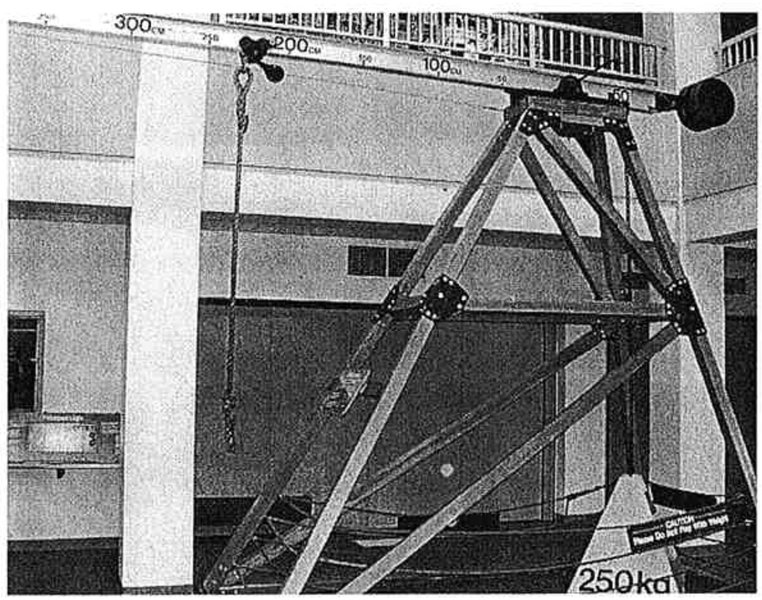




The *Giant Lever* illustrates simple mechanics and mathematics concepts of ratio and proportion.

The Standards that correspond are

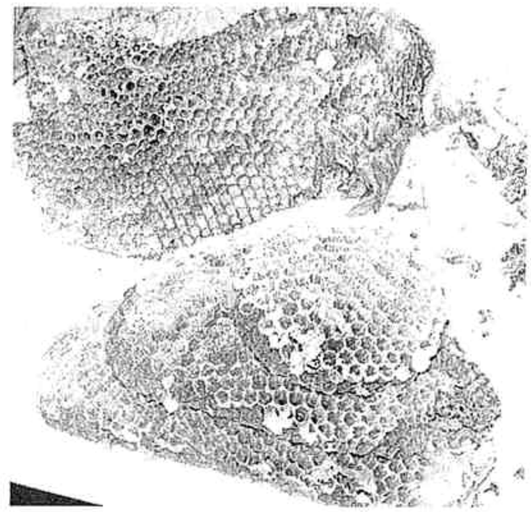
- Standard 4 – Measurement (item 4.2)
- Standard 6 – Problem Solving (items 6.1, 6.2, and 6.3)

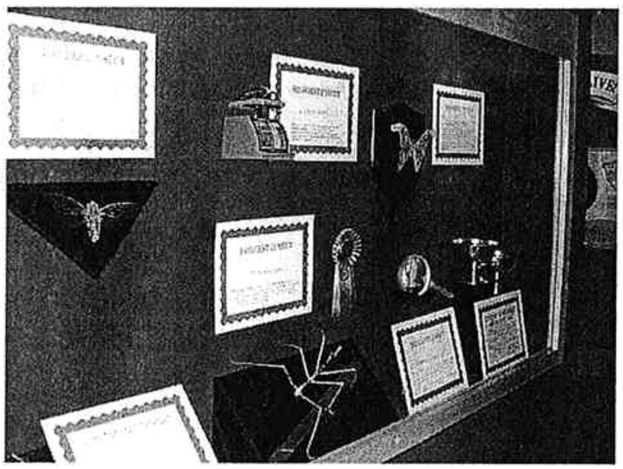


Life Sciences

In Life Sciences, insects provide a great opportunity for including mathematics. The obvious geometric shape of honeycombs built by bees and wasps is a good starting point for mathematical investigations of the natural world. The *Wasp Hives* (right) show repeats of hexagons, making a strong Standards connection:

- Standard 3 – Geometry and Spatial Sense (item 3.1)





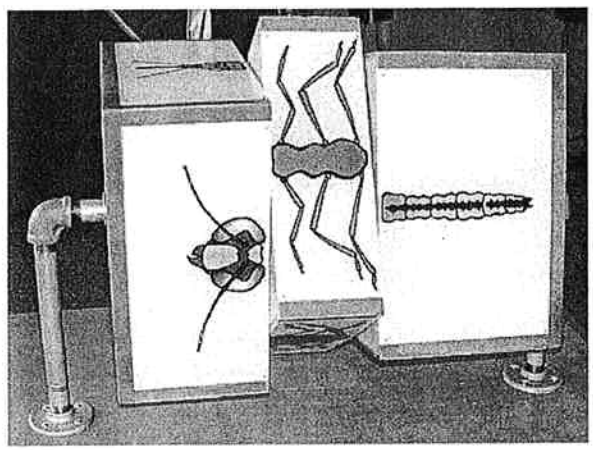
Measurement is frequently used in describing plants or animals. Size, weight, and height are the focus of the wall case at left, which corresponds to

- Standard 4 – Measurement (items 4.1 and 4.2)

Scale and proportion are investigated with fleas and ants, below, and connect to

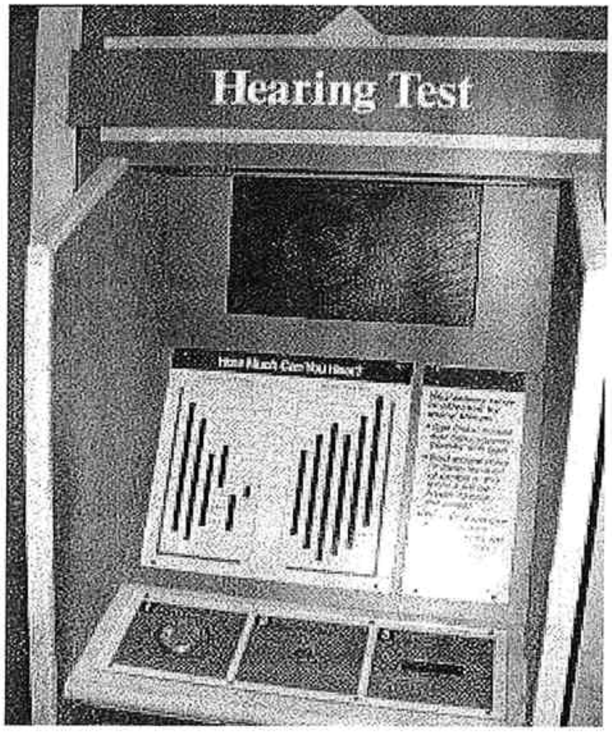
- Standard 1 – Numbers and Operations (item 1.3)
- Standard 4 – Measurement (items 4.2and 4.2)

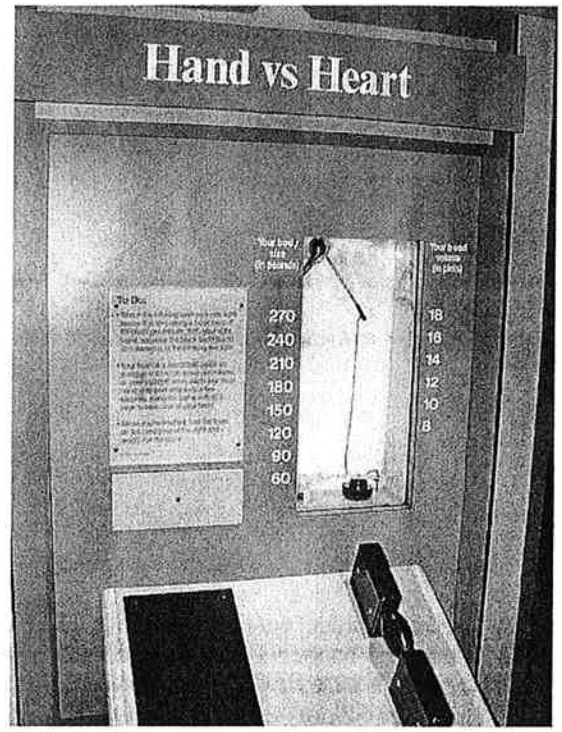
As children move the head, thorax and abdomen sections in *Invent Your Own Insect,* they develop an understanding of finding combinations. Whenever, a head, thoracic, or abdominal section is changed, a new bug is created. This activity is appealing to all ages, but it is fun for older children (ages 8 and up) to try to work out how many possible bugs they can create. They could also discuss the probability of creating a particular bug.



- Standard 1 – Numbers and Operations (item 1.2).
- Standard 5 – Data Analysis, Statistics and Probability Standard (items 5.1. and 5.4)

Human Anatomy





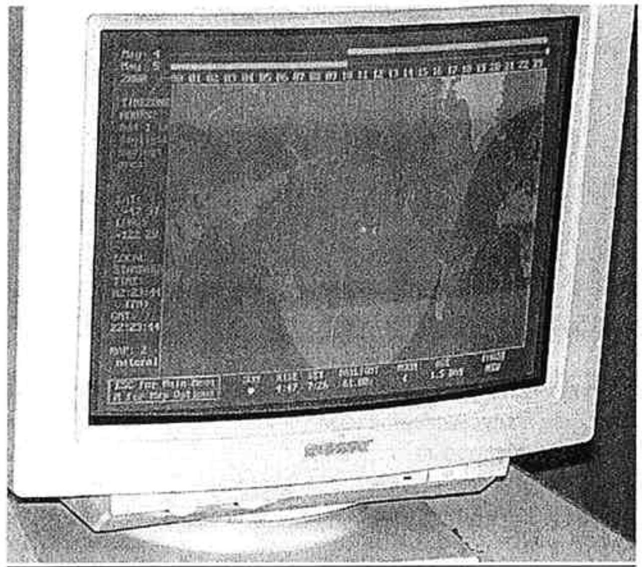
The *Body Works* exhibition provides lots of opportunities for visitors to interact with exhibits and measure personal performance on tasks. The visitor can compare his or her best with averages among different age and gender groups.

- Standard 4 – Measurement (items 4.1.and 4.2)
- Standard 5 – Data Analysis, Statistics and Probability (items 5.1, 5.1, and 5.2)

Earth Sciences

Maps illustrate a number of mathematics connections. For example, this computer screen shows a cylindrical projection map. The image raises lots of questions: What happens to shapes undergoing these transformations? Why does Greenland look so big on this map, but not on a globe? If airplanes essentially fly in straight lines, why do the paths appear as curves on a map? Finally, the twilight line appears as a sine curve rather than a straight line. The related Standard items are:

- Standard 3 – Geometry and Spatial Sense (items 3.3 and 3.3)



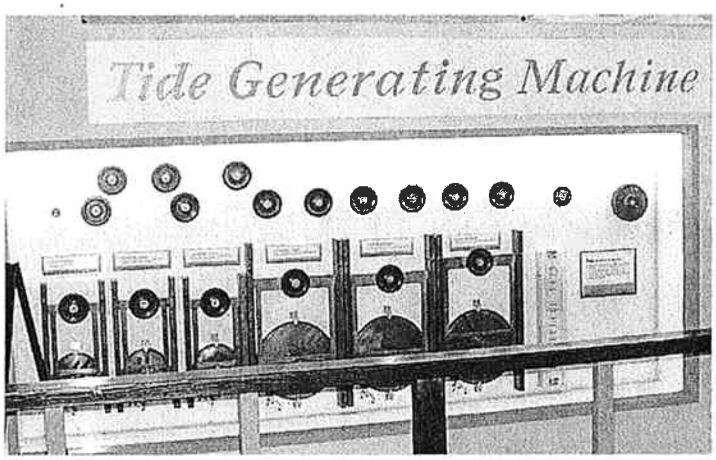
Oceanography may apply only to science centers near coastal regions, but its concepts have solid mathematics potential. At the *Puget Sound Model*, visitors have a scaled-down, birds-eye view (not shown here) of the body of the sound. The model is controlled by a tide-generation component (below left), which displays concepts of wave interference (constructive and destructive), while the 11 "rivers" flow into the model at different flow rates (below right).

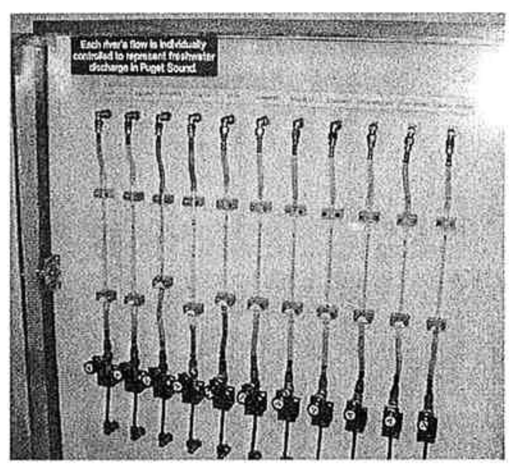
The *Tide Generating Machine* connects with

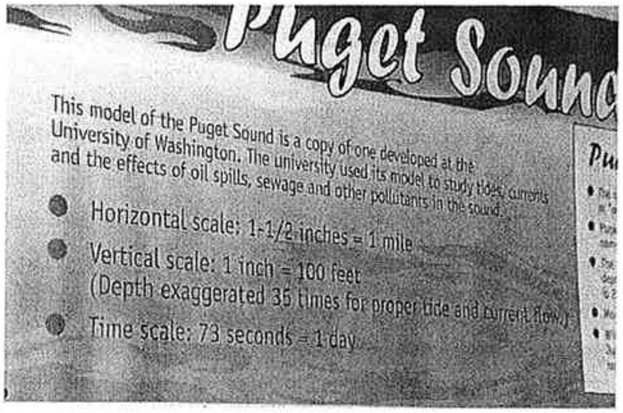
- Standard 2 – Patterns, Functions, and Algebra (item 2.3)

River Flow control panel matches

Standard 2 – Patterns, Functions, and Algebra (item 2.3)
- Standard 4 – Measurement (item 4.1)

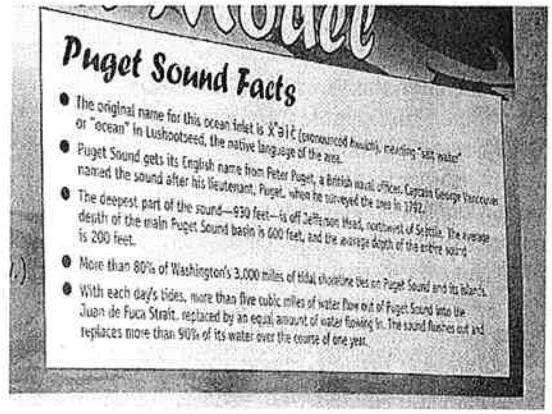






The exhibit copy for the *Puget Sound Model* describes time and space scales used for the model which correspond to

- Standard 1 – Numbers and Operations (items 1.3)
- Standard 4 – Measurement (items 4.1 and 4.2)



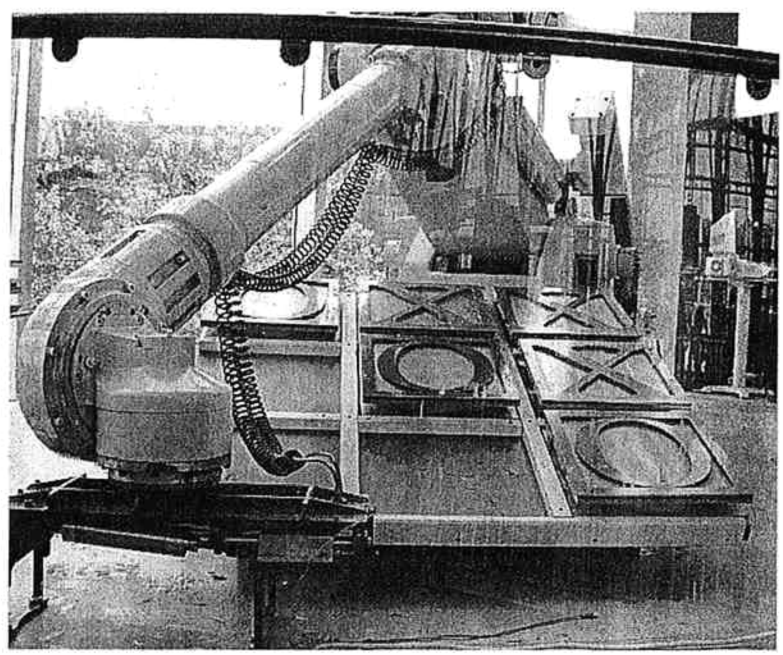
The *Puget Sound Facts* includes numbers, statistics, and percents corresponding to.

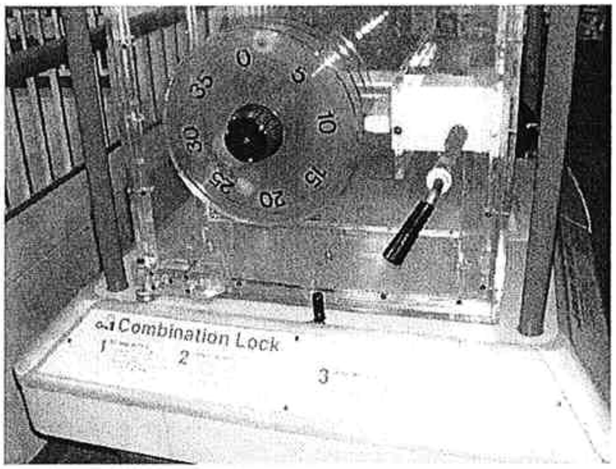
- Standard 1 – Numbers and Operations (item 1.2)
- Standard 4 – Measurement (item 4.1)

Technology

Robots are a natural in a science centers, and this robot plays tic-tac-toe. Strategy and logical reasoning are central elements of this exhibit.

- Standard 2 – Patterns, Functions, and Algebra (item 2.1)
- Standard 6 – Problem Solving (item 6.3)





But technology need not be overly sophisticated to connect with mathematics. A simple combination lock demonstrates combinatorics—a problem of numbers. Locks need a key to line up all the internal components, but in a combination lock a series of numbers is used to line of the slots on the wheels. According to the exhibit copy, "a lock with 30 numbers on the knob can have 27.000 combinations of 3 numbers."

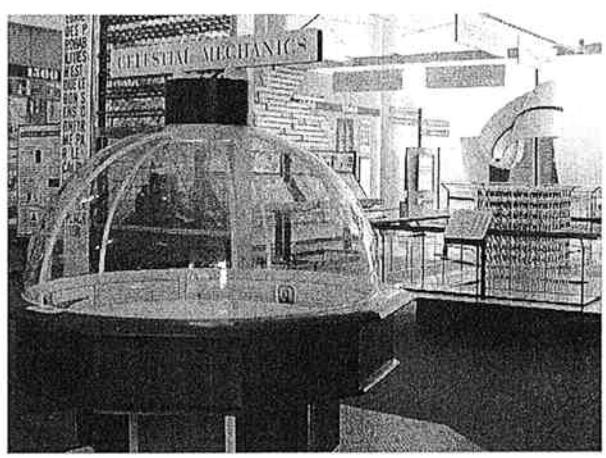
- Standard 1 – Numbers and Operations (item 1.2)
- Standard 5 – Data Analysis, Statistics, and Probability (item 5.4)

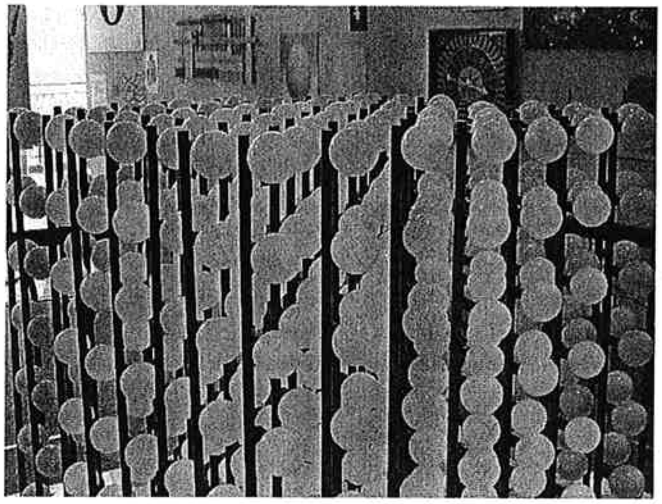
Mathematica—A World of Numbers and Beyond

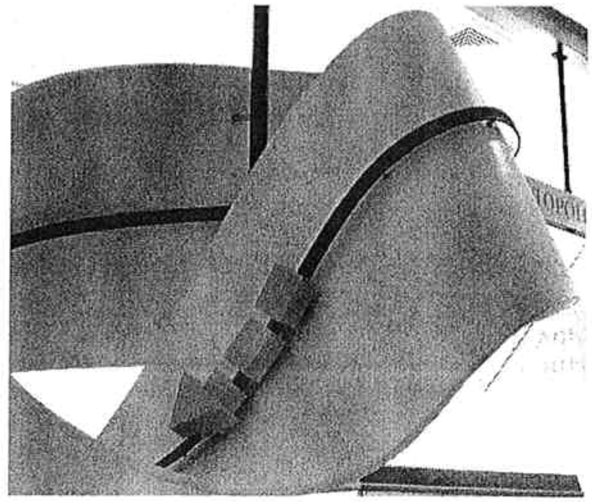
Viewing *Mathematica—A World of Numbers and Beyond* was an unexpected and appreciated opportunity at Pacific Science Center. This venerable exhibit is the one that science center professionals usually talk about when asked if science centers do mathematics. It was created by Charles and Ray Eames approximately 30 years ago, and copies of it were on display at multiple science centers, including the California Museum of Science and Industry, Museum of Science in Boston, Museum of Science and Industry in Chicago, and Pacific Science Center. The exhibit has now been refurbished and copied for traveling.

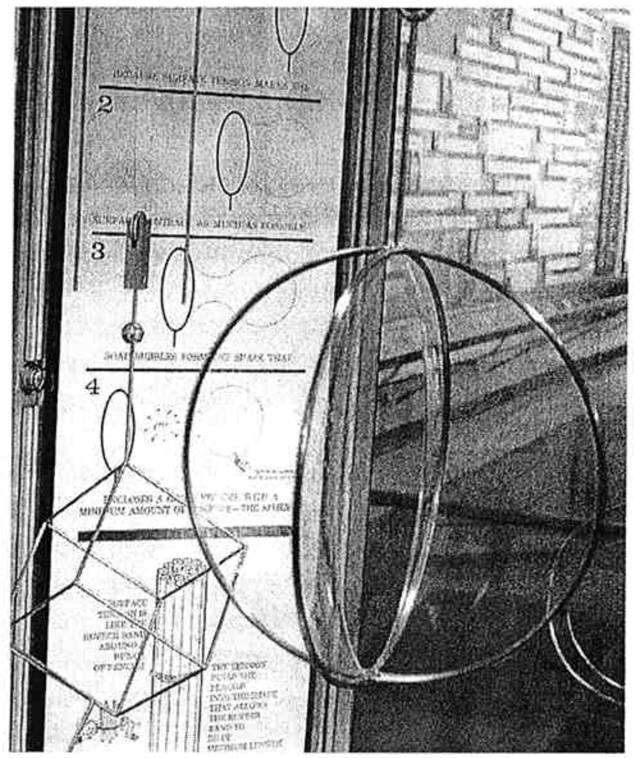
Mathematica presents a fascinating display of mathematics development, and the visitor is introduced to different topics within the discipline. However, it tends to be a "look and read" exhibition, rather than offering the more experiential approaches science centers now take. It benefits from additional activities presented by staff members. For example, when *Mathematica* was on display at Pacific Science Center in the late 1960s and '70s, the exhibits were augmented with tabletop mathematics games and puzzles. A discovery room area provided materials for visitors to make bubble wands in shapes like those of the exhibit hall and Tangram pieces to construct different animal shapes.

A timeline of historic events in mathematics can be found on one wall of *Mathematica,* and throughout the hall there are a number of other interesting exhibits.









The nature of hands-on exhibits at the time *Mathematica* was built required the visitor to push buttons mostly and observe the action. There was little, if any opportunity for the visitor to learn through the manipulation of variables.

The *Calculating-Cube* (above left) provides computed answers by lighting bulbs in the three dimensions of the cube.

The arrow on the *Moebius Strip* (above right) tracks around the surface when the visitor pushes a button.

In the exhibit to the right, visitors push a button to lower different geometric shapes into a tub of soap. The resulting bubbles reflect the science of surface tension and the mathematics of geometric solids

Lessons Learned

Designing developmentally appropriate products and meeting the needs of a widely diverse audience is challenging.

Designing mathematics exhibits for the *Mathfinder* van was not as simple as anticipated. Even though the primary audience is schoolchildren in grades K-8, the intellectual and developmental levels of the children varies. Not only do different audiences want and need different activities, but a free-choice exhibit area must stand on its own and be inviting. There was deep anxiety among staff about how explicit mathematics exhibits should be. Would mathematics terms and experiences draw the visitor or should the mathematics be embedded in nonobvious and nonthreatening ways?

Developmental issues are major design challenges. On the one hand, children respond differently to concepts according to their developmental levels. They have both individual and developmental differences in attention span and responses to frustration. In a pattern activity, young children may want to do dot-to-dot repeatedly. Older children may try a pattern once and move on to new challenges. Staff spoke of these concerns:

> *Mathematics develops sequentially—one concept building on another. Exhibits usually stand alone, or at best, tie topically or conceptually. Rarely do exhibits connect sequentially with one another. Do children "get it" if the experience isn't comprehensive in developing a concept? How do science center professionals know if and when the visitor does "get it?"*

People may not perceive the presence of mathematics when it does not appear in a traditional form they recognize. They ask: Where is the math?

Two different views on this question were found. Much depended on who the audience was. The design team found that volunteers felt uncomfortable if the mathematics was not explicit. Parent volunteers manage the kids in the exhibit hall, and because they are conscientious and want to help kids understand the copy or do the activity, they wanted to see mathematics displayed. They wanted the explanations and the formulas. While piloting exhibits, one volunteer came in and said,

> *This isn't mathematics... where is the math? There is nothing in the label to tell me what the mathematics is.*

The opposing point of view was expressed by one of the site-visit team members—a university mathematics educator. His comment was revealing:

> *Forty minutes was not enough time to see all the mathematics in the Pacific Science Center. Everywhere I looked – math! Math! MATH!*

The problem of making mathematics explicit is not limited to a mini-exhibit hall that is staffed with volunteers. Even very sophisticated visitors, such as members of the site-visit team, recognized the challenge. In the words of one Pacific Science Center team member:

> *So, the science center is full of mathematics, and we need not worry, right? Well, unfortunately, most people would not recognize the mathematics in the activities and exhibits. Most of the math was inherent, but not explicit. Some of the mathematics was hidden. How would we get people to see the mathematics in these activities?*

Sometimes mathematical ideas that work as activities do not translate into inviting exhibits.

Something that works well as a classroom activity, even if it can be represented with objects, may not work as well as an exhibit. For example, Moebius strips are cool and interesting mathematics and give hands-on learning experiences of topology. However, with premade strips people are not inclined to just pick them up. They just don't seem to stimulate curiosity. According to the developers who crafted them for the *Mathfinder* van, Moebius strips require an interpreted experience.

Using Standards to create exhibits and programs benefits teachers (and parents) in utilizing the science center

Within the Science on Wheels program, which targets schools, making the mathematics connection is aided by linking activities and exhibits to their corresponding Math EALR. In choosing topics, making decisions about copy, and designing lessons, the mathematics framework gave the science-center staff and the teachers who get the program a common ground for understanding. The team found a way to tie the exhibit or activity with specific statements within the Essential Learnings. And for the sake of the children, they sought to tie the exhibit to real-world experiences as often as possible.

While the EALRs frame the components that go to schools, there is no comparable effort to use them or the NCTM Standards within the main exhibit halls. Pacific Science Center does directly state that mathematics is part of its mission and has made an effort to link with mathematics educators in the community. Since most exhibits were developed prior to the EALRs, tagging exhibits to corresponding Standards might be a mutually beneficial activity for both Pacific Science Center and the Washington State Mathematics Council.

Ann Arbor Hands-On Museum
www.aahom.org

Ann Arbor Hands-On Museum

A Tale of Two Exhibitions and Their Exhibits

The site visit to Ann Arbor Hands-On Museum, Ann Arbor, Michigan, permitted the team to see two exhibitions: Ann Arbor's frequently-traveled *Geometry in Our World* exhibition and *Fun, 2,3,4: All About A Number of Things,* an exhibition created by the Sciencenter, Ithaca, New York. We also heard about Ann Arbor's new *Solve It* exhibition.

Ann Arbor recently brought in a new director, Jim Frenza, who has been challenged with expansion and exhibit-development issues. The museum had recently opened its new space when the site team visited. An expanded area gave toddlers a play space and allowed the traveling exhibition *Fun 2,3,4* ample room for set up. Staff had then turned their attention to developing the oft-delayed *Solve It* exhibition.

Solve It, an NSF-funded project, was still in conceptual stages[20] at the time of the site visit. It had undergone significant revision from the time of initial proposal to the blueprint stage. *Solve It* is to be an exhibition on puzzles and problem-solving approaches. Rather than use existing (and consequently well-known) puzzles, the development team was working to create new items. Their intent is to engage the visitor in applying different strategies to obtain a solution.

The two mathematics exhibitions at Ann Arbor during the time of the site visit are the focus of this case study. *Geometry in Our World* is a permanent exhibition at Ann Arbor, with several copies available for traveling. It is an exhibition about shapes: polygons, polyhedra, and Tangrams. *Fun 2,3,4: All About A Number of Things* is an exhibition about counting, measuring, and graphing.

Mathematics at the Museum

Geometry in Our World

As the teachers' guide to the *Geometry in Our World* exhibition notes:

> *Geometry comes from words meaning "world" and "measurement." [It] arises from humankind's need to describe its surroundings. Many natural objects can be best understood as...geometrical concepts. Some striking*

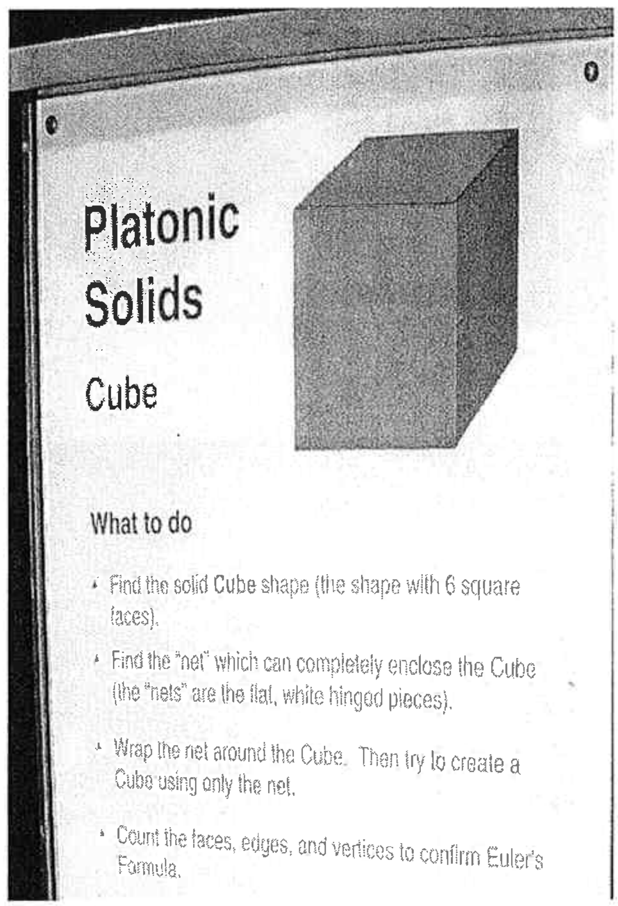
[20] *Solve It* opened June 2001. ASTC will help travel this exhibition, starting in 2002.

examples are shells...flowers and animals...soap bubbles and planets... .

Human-made objects are intimately related to the ideas: buildings, bridges...sculpture and painting...games and puzzles. The latest uses of geometry are provided by increasingly sophisticated computer tools.

The *Geometry in Our World* exhibition conceived under the leadership of Cynthia Yao, founding director of the Museum, provides examples of these geometrical objects and concepts. Some invite the visitor to manipulate objects to solve problems and to illuminate mathematical concepts. Some ask the visitor to read and conceptualize ideas. Others simply exist as awe-inspiring examples of geometrical concepts. In all of the exhibit elements, the developers paid attention to the NCTM Standards and the Michigan mathematics standards, including the Professional Teaching Standards[21] concerning the choice of tasks, teacher and student roles, and the use of tools.

Polygons and polyhedra form a major section of the *Geometry in Our World* exhibition. As a collection of objects, polygons start with points and lines: for example, squares and triangles. If, when looking at objects bounded by polygons, the object appears three-dimensional (such as cubes or pyramids), it is called a polyhedron. The exhibition provides both hands-on components and text.



A kiosk provides a generic description of the five Platonic Solids. Each is a solid figure with outside faces made up of the same regular polygons, with faces, edges, and vertices (corners) exactly alike.

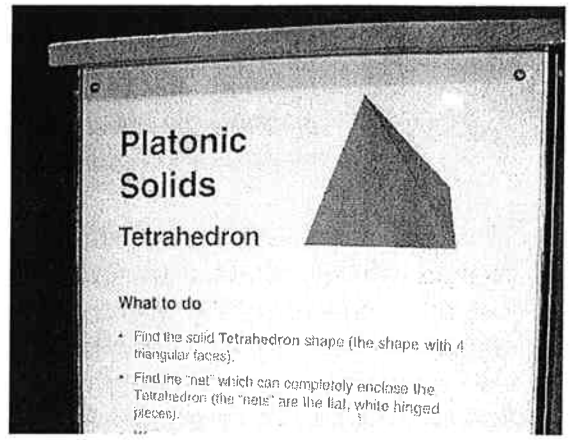
The *cube*, shown to the right, is familiar to most visitors and introduces the idea of faces and vertices. The cube has six square faces, with three squares meeting at each vertex.

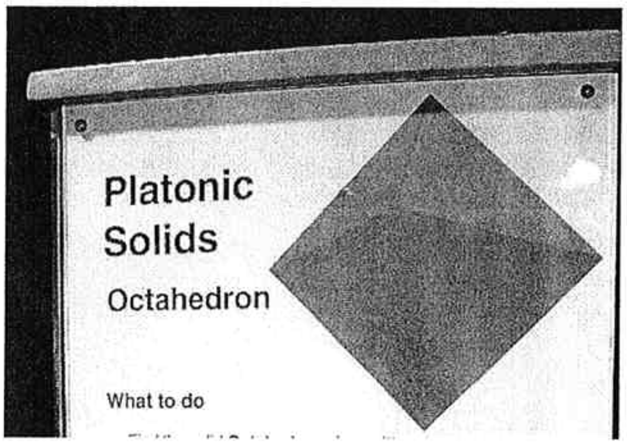
The applicable NCTM Standards are

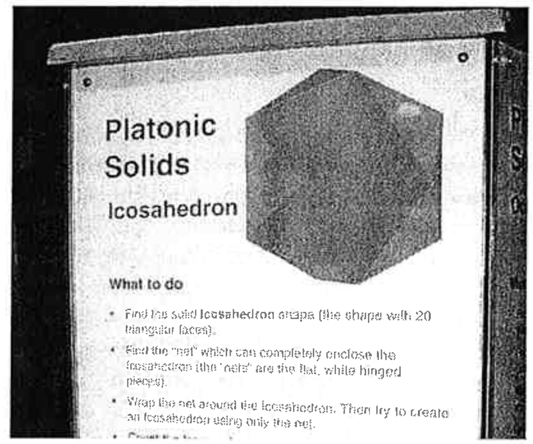
- Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Patterns and Functions (items 2.1, 2.2, and 2.3)
- Measurement (items 4.1 and 4.2)
- Connections (items 9.1 and 9.2)
- Representation (item 10.3)

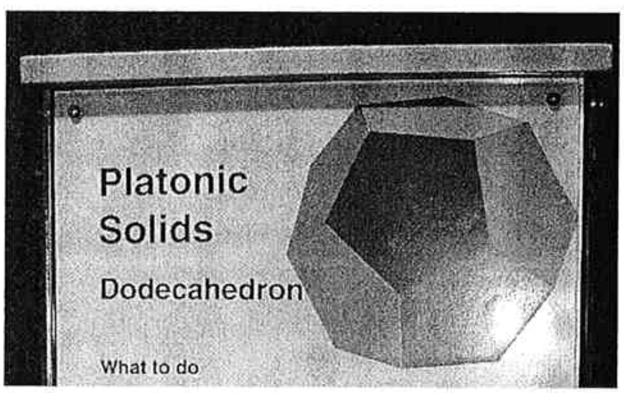
[21] While the draft NCTM Standards, 2nd Edition, are included in the Appendix, the Professional Teaching Standards for Mathematics are not. Please refer to the NCTM web site, www.nctm.org, for more information.

A *tetrahedron* is built from triangles, as is the *octahedron* (below left) and the *icosahedron* (below right).



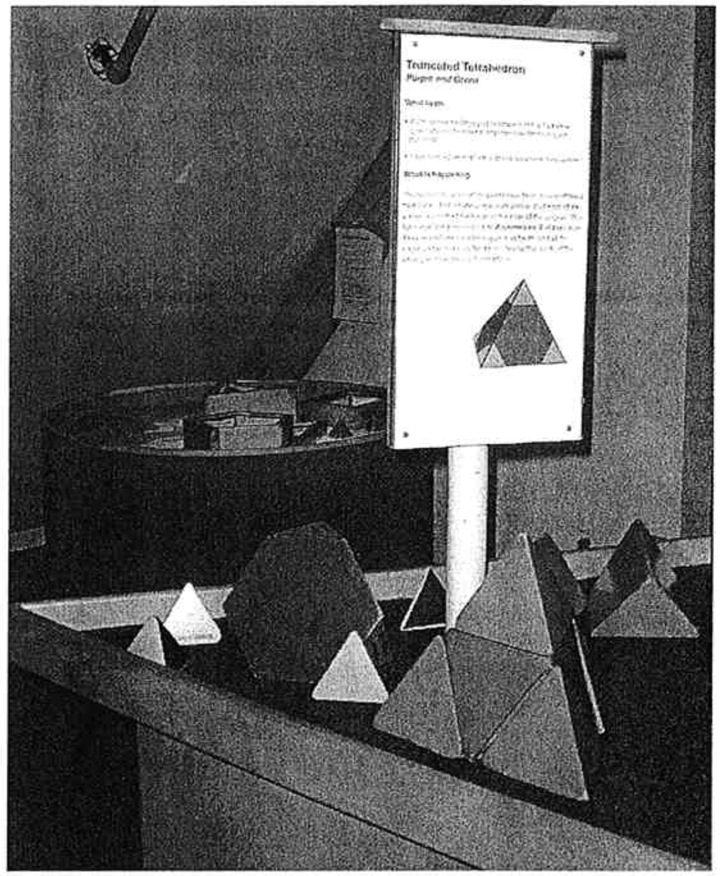






The *dodecahedron* (shown at left) is built from pentagons—five-sided shapes. Each sign asks the visitor to find the "net"—flat, white hinged pieces—which can be folded to enclose the object. Then the visitor is told to count the faces, edges, and vertices to confirm Euler's Formula, which is also described on the kiosk.

Visitors get a chance to "break apart" the solids they read about on the kiosk. In the exhibit at right, the tetrahedron is truncated. The directions tell the visitor to put together the large purple shape and the four small green shapes to make a large tetrahedron (triangular pyramid), then take them apart and look at the faces where they connect.

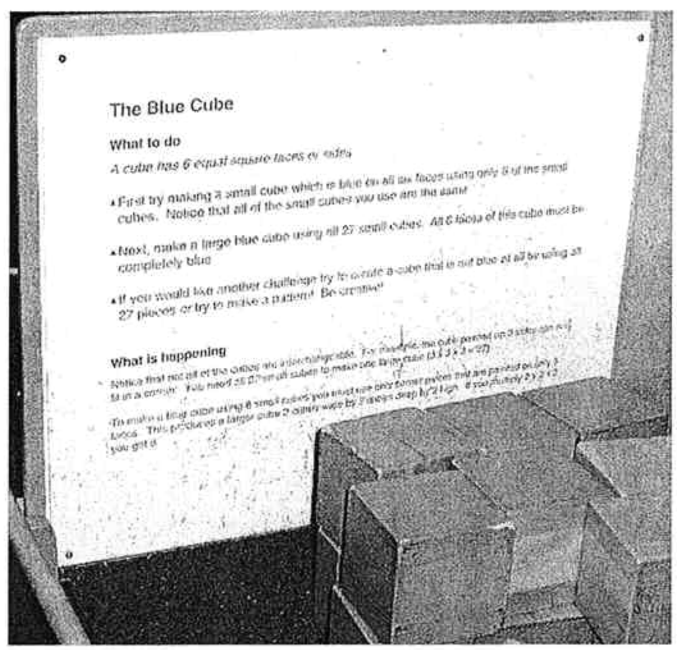


The copy explains that "truncated" means that the points have been sliced off like a tree trunk. It labels the polyhedron as an Archimedian Solid, and directs the visitor to notice that each of the small green pieces is a tetrahedron.

The corresponding NCTM Standards are

- Standard 2 – Patterns and Functions (items 2.2 and 2.3)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.2, 3.3, and 3.4)
- Standard 4 – Measurement (item 4.1)
- Standard 6 – Problem Solving (items 6.1, 6.2, and 6.3)
- Standard 9 – Connections (items 9.1 and 9.2)
- Standard 10 (items 10.1 and 10.3)

Another closely related experience permits the visitor to break apart one large cube, which is painted on the "outside," into multiple cubes. There are challenges, such as building a 3x3x3 uncolored cube or building a 2x2x2 colored cube.



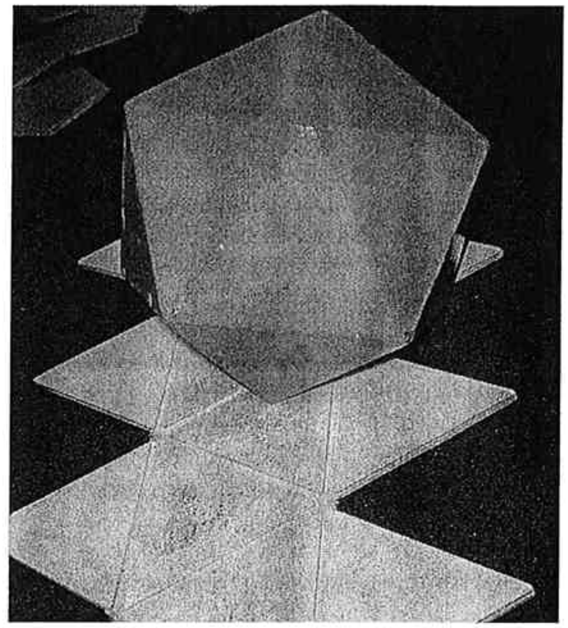
NCTM connections are

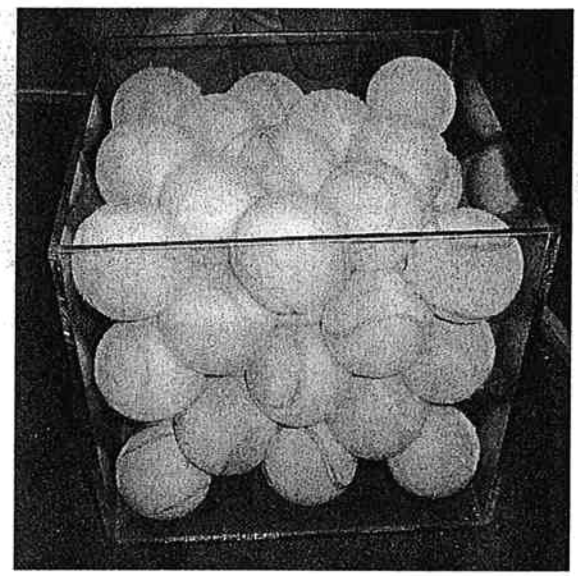
- Standard 1 – Numbers and Operations (items 1.2 and 1.3)
- Standard 2 – Patterns and Functions (items 2.1 and 2.3)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 4 – Measurement (items 4.1 and 4.2)

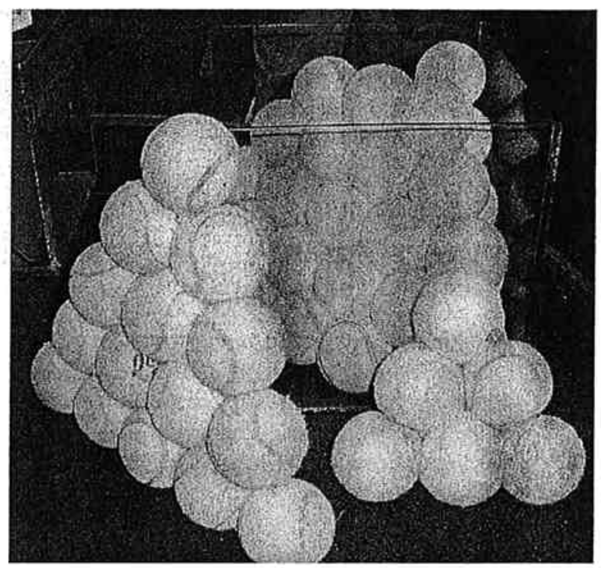
With this exhibit, the visitor sees how a two-dimensional "net" can be folded into a three-dimensional object. The exhibit reinforces the reasoning that connects two and three dimensions. Younger children can explore folding the nets into the corresponding three-dimensional objects. Middle school students can tackle making the nets for three-dimensional objects.

The exhibit reflects

- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)





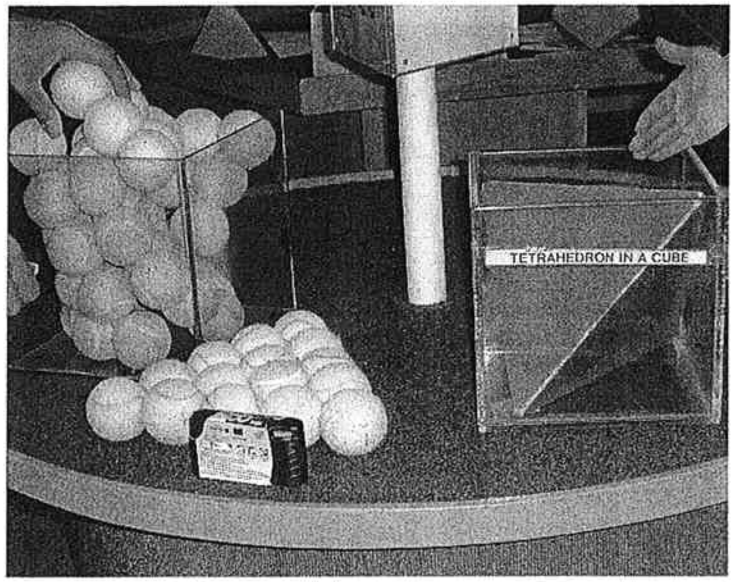


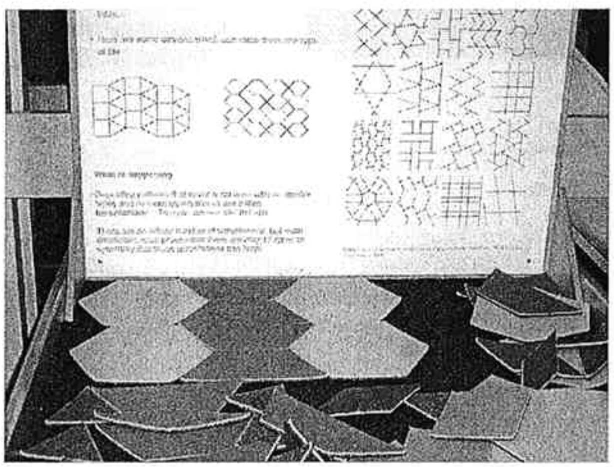
Starting with a clear plastic box filled with tennis balls, the visitor quickly discovers that the balls are fused into triangular shapes. Once the clusters are taken out of the box, the challenge is to replace them.

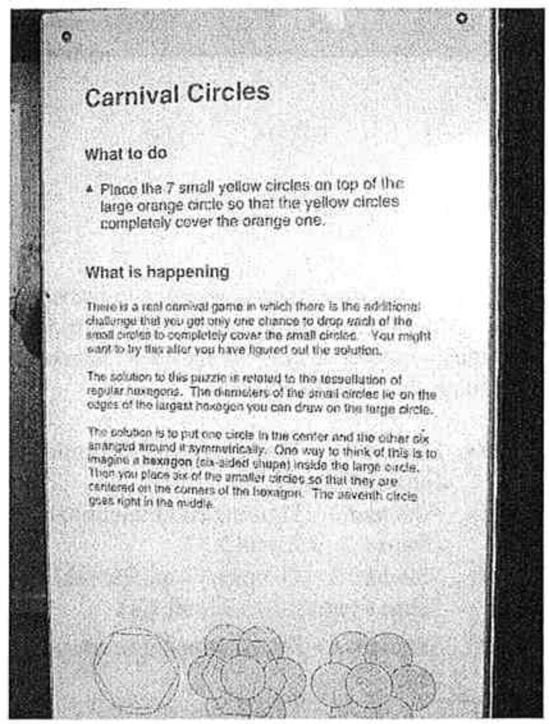
The applicable Standards are:

- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 6 – Problem Solving (items 6.1, 6.2, and 6.3)

The hint is provided in the model sitting to the right, which includes a tetrahedron in a cube.



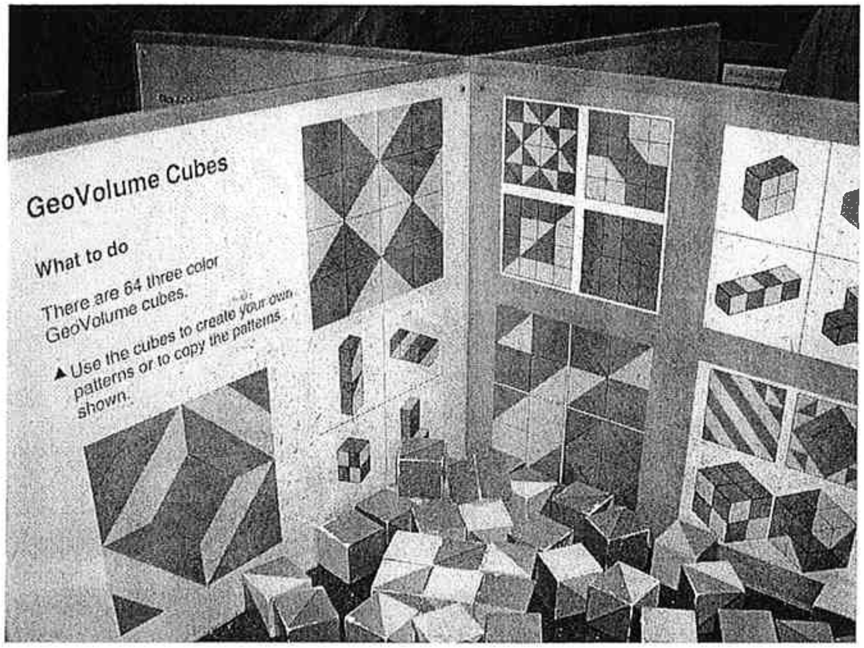




Geometry in Our World gives plenty of practice with two-dimensional shapes and patterns. One exhibit invites visitors to make tessellating patterns, which, according to the copy, are repeating patterns that cover a flat area with interior holes and no overlapping pieces. The other exhibit (shown at right) challenges visitors to cover a large circle with as few small circles as possible.

Standards corresponding with these two activities are

- Standard 2 – Patterns and Functions (items 2.1 and 2.2)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 6 – Problem Solving (items 6.1 and 6.3)



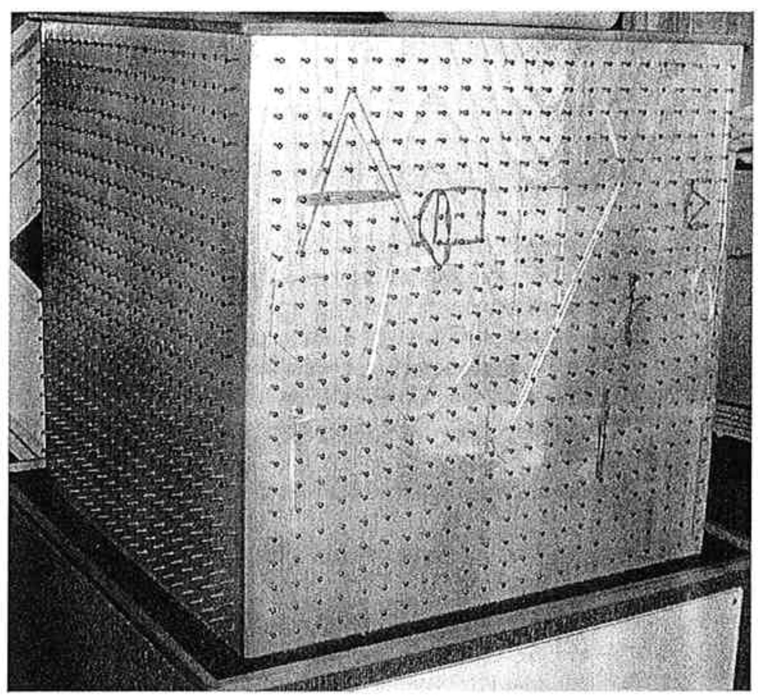
The visitor can also build designs using cubes, which the copy encourages the visitor to do.

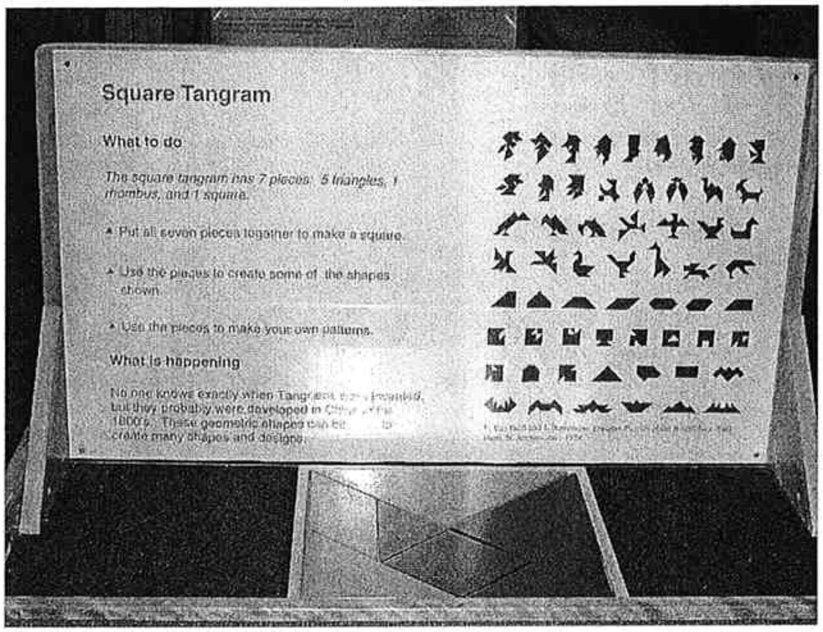
Corresponding NCTM Standards include

- Standard 2 – Patterns and Functions (items 2.1 and 2.3)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 6 – Problem Solving (items 6.1 and 6.3)
- Standard 10 – Representation (items 10.1 and 10.3)

Plane geometry is also presented in discovery fashion. Many visitors and students discover such concepts as the Pythagorean Theorem using the Geoboard Cube.

- Standard 1 – Numbers and Operations (items 1.2 and 1.3)
- Standard 2 – Patterns and Functions (items 2.1, 2.2, and 2.3)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 4 – Measurement (items 4.1 and 4.2)

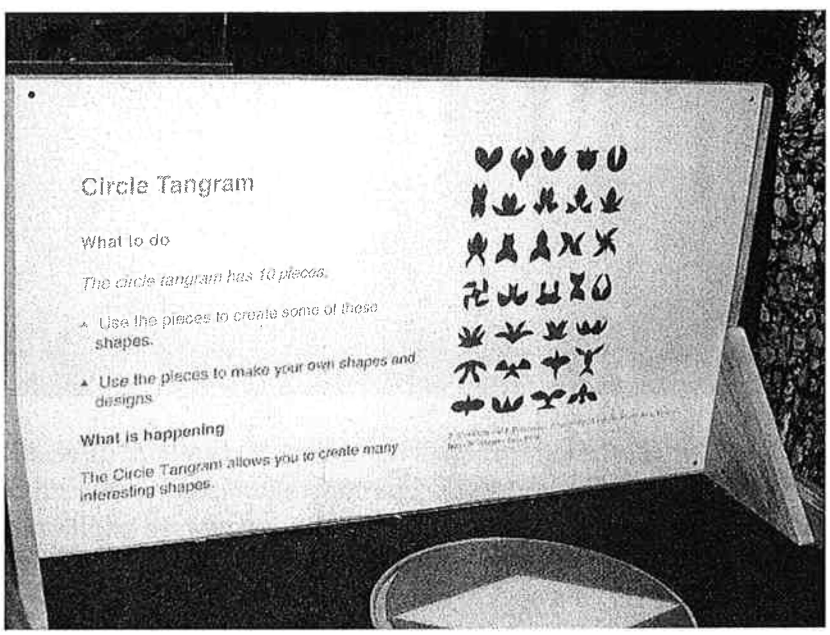




Two different puzzle activities let the visitor explore the polygon shapes in the *Square Tangram* (above) and the patterns resulting from the *Circle Tangram* (below).

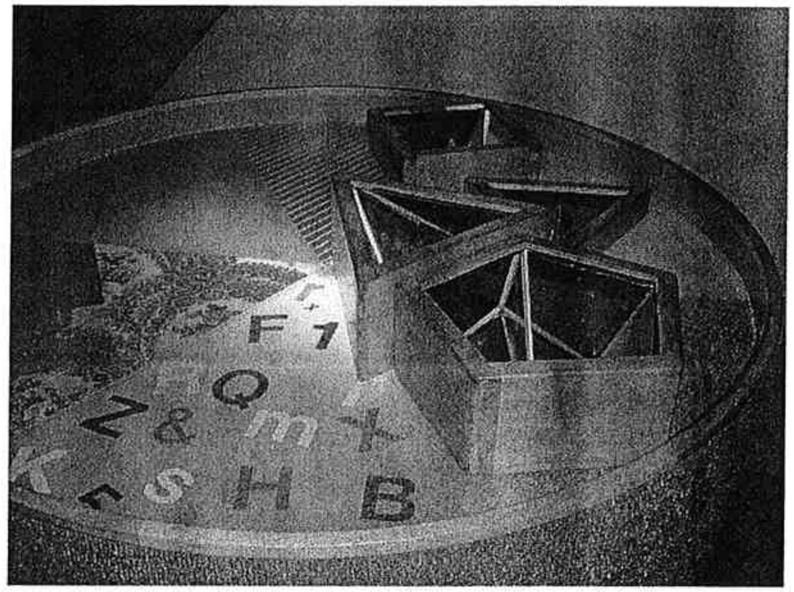
Both puzzles reflect Standards items:

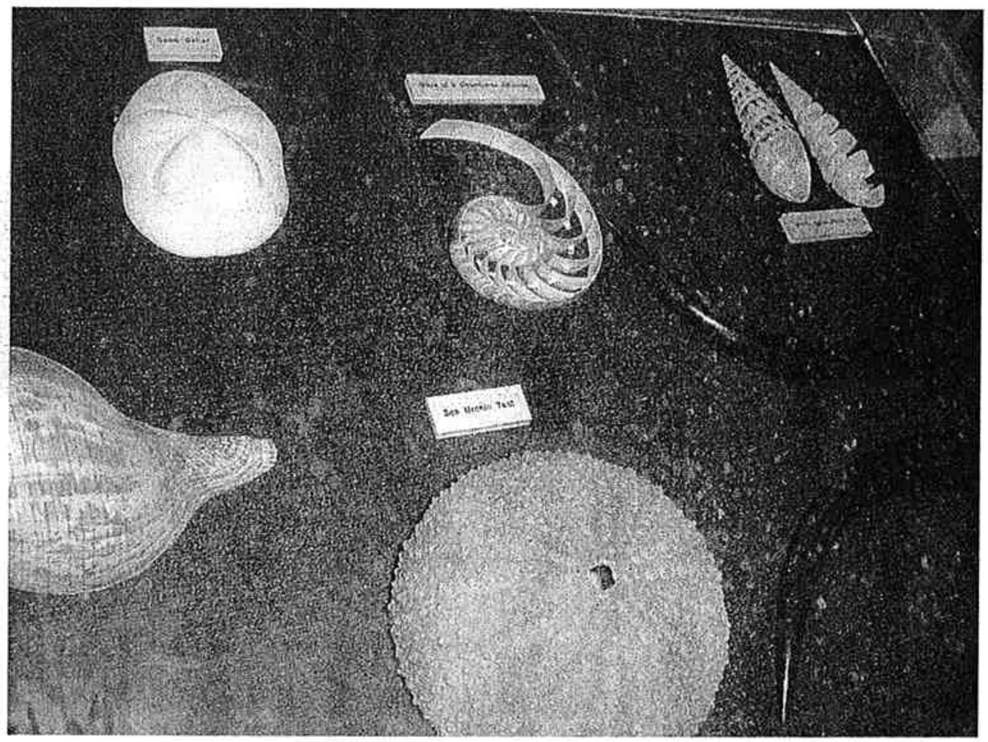
- Standard 2 – Patterns and Functions (item 2.1)
- Standard 3 – Geometry and Spatial Sense (items 3.1, 3.3, and 3.4)
- Standard 6 – Problem Solving (items 6.1 and 6.3)



Mirrors in polygon-shaped boxes let the visitor develop ideas about symmetry. The concepts are then related to real-world objects, such as shells.

- Standard 3 – Geometry and Spatial Sense (item 3.3)





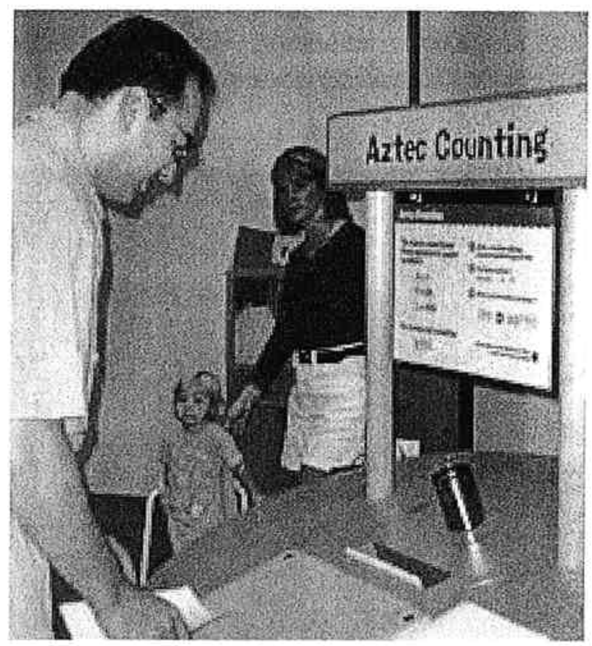
The *Geometry In Our World* exhibition is old and well used, but quite effective in supporting visitors in learning concepts. The copy associated with each component helps the visitor explore the activities and understand the concept. In addition, there is a Teacher's Guide, which expands the activities for classroom use and further explains the concepts.

Fun, 2,3,4: All About a Number of Things

Fun 2,3,4 started when the Sciencenter was a storefront museum in Ithaca. Initially calling it "*Counting on You*," the museum offered the components as prototype exhibits on counting. When the Traveling Exhibits at Museums of Science (TEAMS) consortium was formed, Charlie Trautman, the director of Sciencenter said, "We have these prototypes and can do a mathematics exhibition."

The prototyping process allowed the developers to observe visitor reactions for a significant period of time. In this case, the museum spent 2½ years testing exhibit components. But exhibits tested for a permanent exhibition do not necessarily translate into items for a traveling exhibition. Thus, when the collaborative was formed, Sciencenter used the support from NSF to develop the standards for exhibitions that would travel. Changes were then incorporated into the *Fun 2,3,4* exhibition.

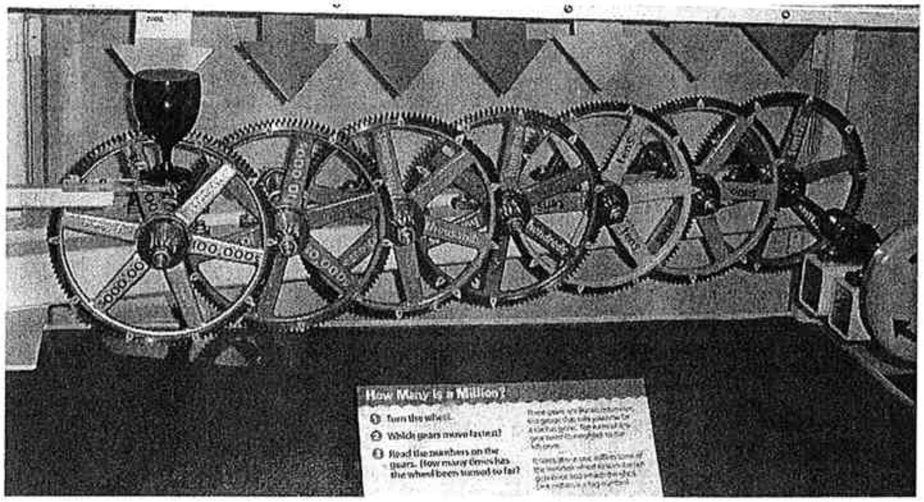
Ultimately, the design team settled on three concepts to address in the exhibition: Counting, Measurement, and Graphing.



Several exhibits in *Fun, 2, 3, 4* help the visitor think about what it means to "count." Exhibits, such as Aztec Counting (to the right) help visitors see different cultures deal with counting.

Although the NCTM Standards were not used to construct the exhibition, exhibits can be matched to the specific items. Aztec Counting corresponds with Standards items:

- Standard 1 – Numbers and Operations (items 1.1, 1.2, and 1.3)
- Standard 2 – Patterns and Functions (item 2.1)
- Standard 4 – Measurement (item 4.1)
- Standard 10 – Representation (items 10.1 and 10.3)



How Many Is a Million? corresponds to

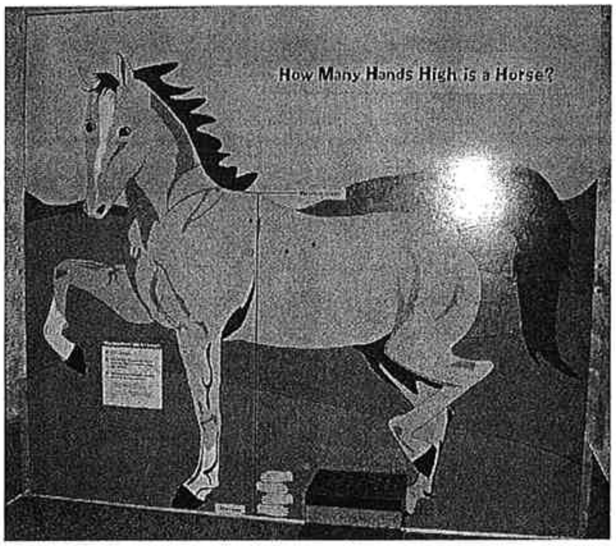
- Standard 1 – Numbers (item 1.1) and
- Standard 4 – Measurement (item 4.1)

How Many Is a Million? was uninteresting to visitors until the exhibit was modified. Initially, visitors were asked to turn a crank that rotated gears and ultimately counted to 1 million. Essentially, visitors looked at it, tried the crank one or twice, and left. Motivation came when staff placed a wine goblet in such a way that when the count reaches 1,000,000, it shatters. Visitors at first believe they will be the ones to break the glass. Soon they discover how big a million is!

Counting (adding and doubling) forms the basis for the *Double My Allowance* exhibit. Corresponding NCTM Standards include

- Standard 1 – Numbers and Operations (items 1.1, 1.2, and 1.3)
- Standard 2 – Patterns and Functions (item 2.1)
- Standard 4 – Measurement (item 4.1)
- Standard 5 – Data Analysis and Probability (items 5.1 and 5.3)
- Standard 9 – Connections (items 9.1 and 9.3)

Closely related to the counting area, measurement applies counting strategies to understanding "How much?" or "How big?" The following exhibits ask the visitor to make measurements using nontraditional standards.



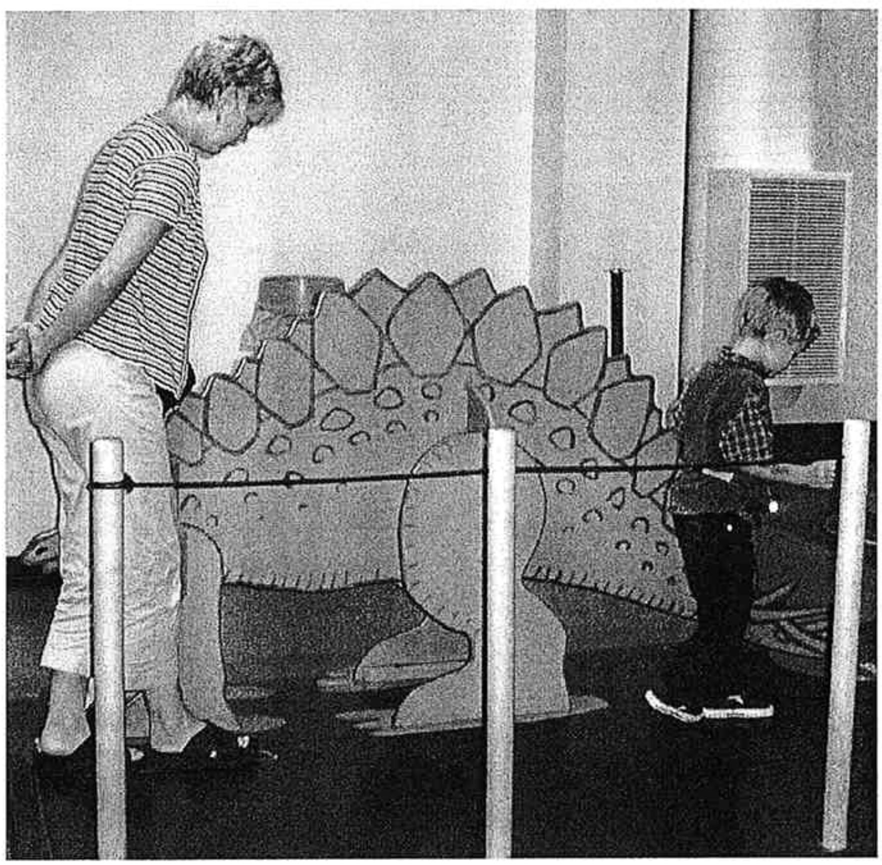
How Many Hands High Is a Horse? asks the visitor to measure the height of a horse by counting the number of hands from floor to the horse's withers.
The Standards connections are:

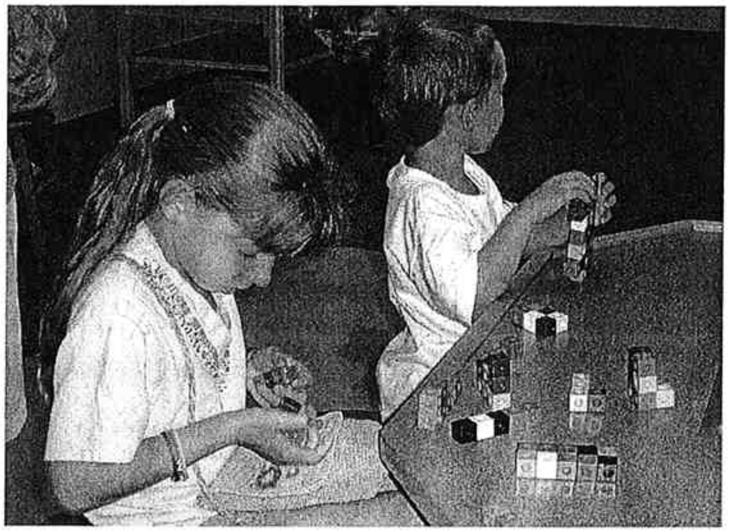
- Standard 1 – Numbers and Operations (items 1.1 and 1.3)
- Standard 4 – Measurement (item 4.1)

Children walk beside the dinosaur, measuring its length in "kid feet." Very young children have the opportunity to measure using nonstandard units. For older children and adults, the activity opens the door for them to ponder the inverse relationship between the size of the unit (a person's foot) and the number of units needed to measure a set length.

The Standards applied are

- Standard 1 – Numbers and Operations (items 1.1 and 1.2)
- Standard 4 – Measurement (item 4.1)



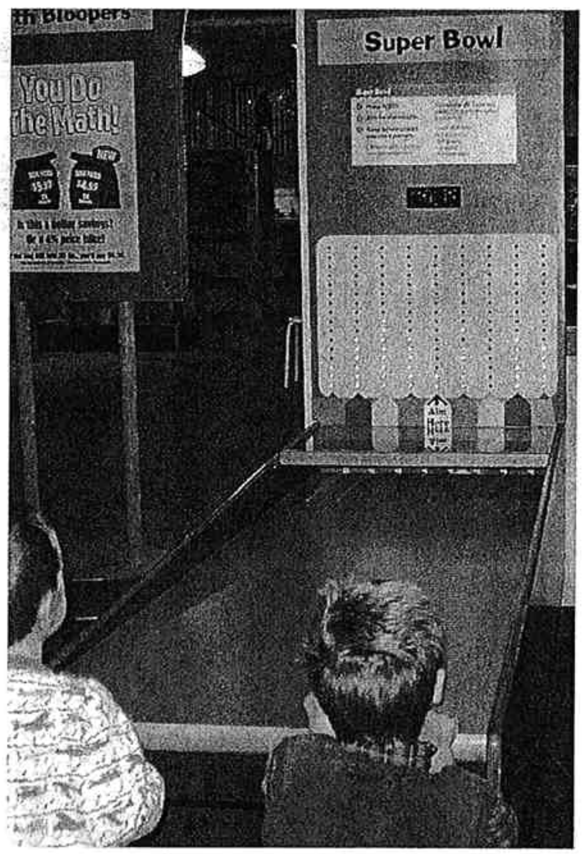


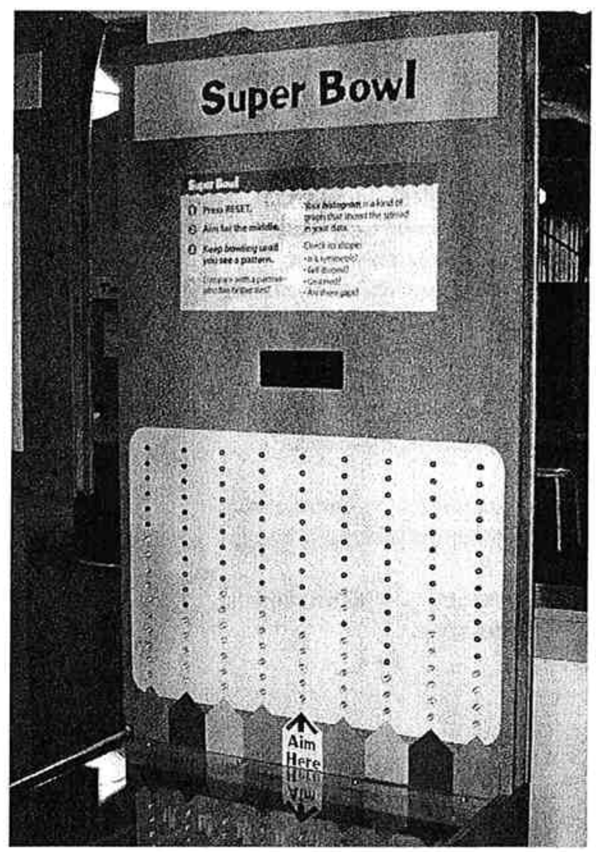
Double the Doggie asks visitors to double the size of the sample plastic-cube dog. It requires that the visitor consider what it means to expand the size of an object in three dimensions.

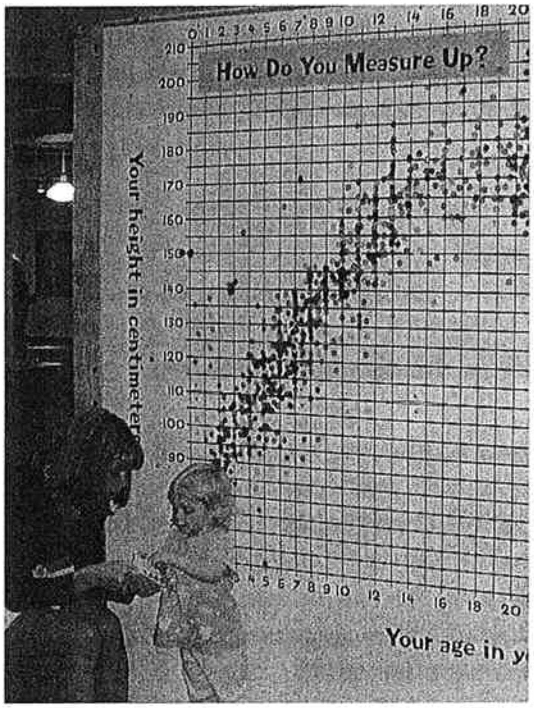
Standards applied are:

- Standard 3 – Geometry and Spatial Sense (items 3.1 and 3.3)
- Standard 4 – Measurement (items 4.1 and 4.2)

Counting teams up with graphing in the *Super Bowl* exhibit. The visitor rolls a ball down the alley. Where it enters the end point, the mark is scored and graphed. Lights indicate how frequently the ball hits that particular spot.





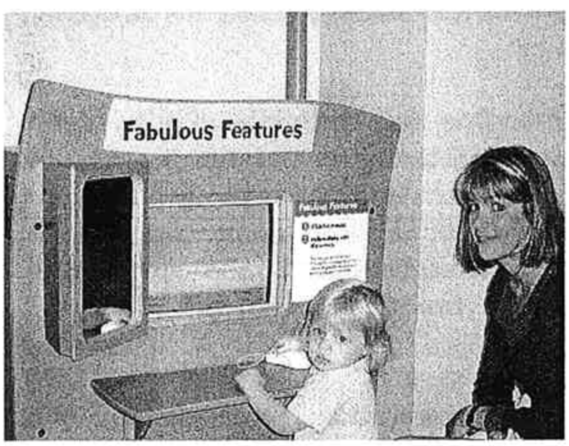


Some of the more popular exhibits are those with graphing components. Visitors of all ages spent time at these exhibits.

In How Do You Measure Up? visitors have a chance to compare the pattern that results when each person records his or her height and age in a single point on a coordinate graph. Even young children can see the trend as the number of data points increase. Older children and adults can discuss how the graph is made and the appearance of a correlation between the two measurements.

The graphing components address the following mathematics standards:

- Standard 1 – Numbers and Operations (items 1.1 and 1.3)
- Standard 2 – Patterns and Functions (items 2.1 and 2.2)
- Standard 3 – Geometry and Spatial Sense
- Standard 4 – Measurement (items 4.1 and 4.3)
- Standard 5 – Data Analysis, Statistics and Probability (items 5.2 and 5.3)



Evaluation

Not all exhibitions are evaluated. However, Mark St. John of Inverness Research Associates conducted a summary evaluation of *Fun 2, 3, 4.* The findings from this evaluation are published on the Sciencenter's web page at http://www. Sciencenter.org.

Lessons Learned

People use exhibits as they want, not necessarily as the designer intended.

Despite every good intention of an exhibit designer, people use exhibits in the manner that most intrigues or pleases them. This may, in fact, be counter to the intention of the designer and contradictory to the exhibit copy. The difficulty comes when the visitor mistakes the consequences of his or her actions for the concept the copy articulates. For example, in *Super Bowl* most of the young people using the exhibit sought to spike each of the lines of lights. The resulting rectangular block of lights does not exemplify the intended bell-shaped curve. The potential for misconception exists.

What should be done about an exhibit that shows potential for misconceptions? What is the responsibility of the science center when visitors leave with a misconception generated by an exhibit experience? These questions became a serious topic of discussion among the site-visit team and museum staff. Many professionals in science centers are satisfied if visitors engage with an exhibit. From their perspective, the activity will create a "placeholder" in the person's mind, such that when the idea is revisited in a different context, the concept will then make sense.

Other observers have been more critical of exhibits that, from their point of view, don't work. The exhibit may not work for several reasons, including the fact that the user may be too young to understand the underlying concept. Such was the case for the little girl pulling up the tubes in the *Double My Allowance* exhibit. Until her mother came over to intervene, the child simply pulled the tubes up, looked at the contents, and shoved them down again. Also visitors often do not spend enough time with a given exhibit to construct the exhibit's intended meaning.

More problematic are exhibits in which signage misdirects action, is contradictory, or is simply wrong. The ideas conveyed may cause even sophisticated participants to misconstrue the concepts involved. While not pointing to components of either exhibition presented here, these issues are serious enough to warrant an extended discussion among science center professionals.

Does the word "mathematics" cause people to avoid the exhibits?

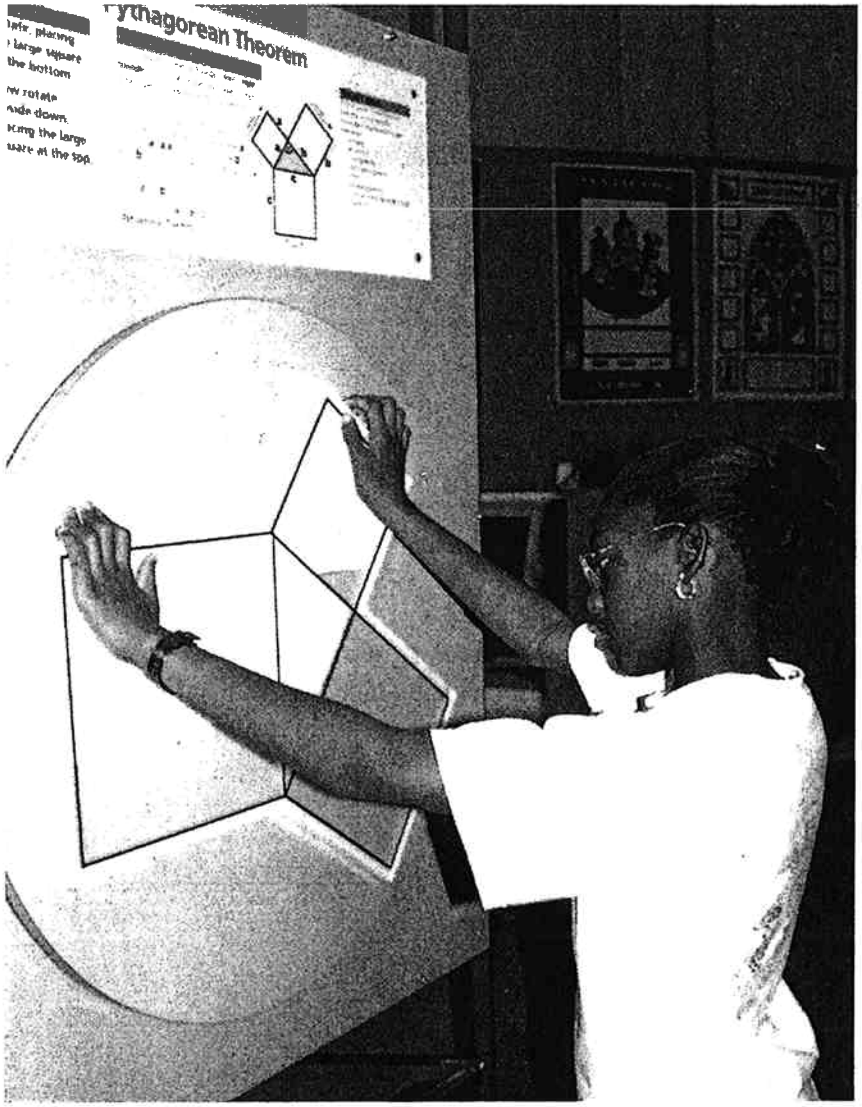
The names of both these exhibitions demonstrate point-counterpoint with an issue that surfaced at every site we visited: Do you tell visitors that the exhibit or exhibition is about mathematics? In developing *Fun, 2, 3, 4,* the Sciencenter felt that using certain words, such as "mathematics" or "arithmetic," would avert interest in the exhibition. They chose the title to be inviting, and clearly there is enough information here to make the mathematics connection. This is not always the case with math exhibits.

On the other hand, Ann Arbor chose *Geometry in Our World* for its mathematics exhibition title. The title seems not to have hurt attendance to the exhibition, but the site-team encountered conflicting attitudes on this issue:

> *If you use mathematics words, people won't come. If you don't use mathematics language, you trick people—people might come and find the exhibit fun. But they may not decipher that they are doing mathematics.*

If a well-used mathematics exhibition is still useful, it should be refurbished, not retired.

There is a tendency among museums to retire exhibits when they show their age. The *Geometry in Our World* exhibition is to be retired, although at this point Ann Arbor Hands On Museum is considering refurbishing its copy with components from the multiple traveling copies. From the content point of view, *Geometry* is still a relevant and useful exhibition. Revitalizing quality mathematics exhibition, such as *Geometry*, is something science centers should consider.



Findings, Emerging Issues, and Recommendations

Findings

The findings grow out of the lessons learned from each of the sites the team visited. Team members debriefed each other after each session, and the insights the team gained were transcribed in a running narrative. Sometimes site-visit team members would prepare a written report following the visit to share additional perspectives. The team asked the professionals at each science center what insights they wished to share, and reviewed some of the mathematics and learning research literature. The emerging research on cognition and learning presented in a 1999 report on learning by the National Research Council (Bransford, Brown, & Cocking, 1999) was especially persuasive. This research has provided new and deeper understandings of how people learn, which has implications for the work of science centers. [22].

Science Centers produce various kinds of mathematics exhibits, programs, and materials that serve multiple audiences.

The science center field has demonstrated the capacity to provide engaging mathematics exhibits, programs, and materials. Five cities were visited by the team to detail the work of six institutions and some team members took side trips for quick looks at other institutions[23]. The lists below note the range of displays, programs and materials found, as well as the various audiences served.

Mathematics in Science Centers

Displays, Programs, and Materials	Audiences
exhibits	preschool
workshops	K-12 school groups
curriculum	parents and families
kits	inservice teachers
trade publications for general use	preservice teachers
web sites	university faculty
programs in schools and classrooms	general public

[22]Although there is more to be considered about how people learn than can be presented here, the research was helpful in thinking about the implications of what was found. In particular, the research findings about transfer of learning—for example, knowing how and when to apply what is learned in one context to a problem in a different context—are relevant to the work of science centers in mathematics. The research on how people learn suggests new ways for mathematics teaching and learning and provides a strong motive for the contribution of science centers.

[23] Dallas' Science Place, COSI Toledo, and Paris' La Cité des Sciences et de l'Industrie. These were opportunistic visits, as they were close to sites selected for this study or accessible because of personal vacation travel.

Science centers add an important experiential component to the formal mathematics instruction provided by schools.

Science centers provide an important experiential component to accompany the formal mathematics instruction provided by schools, although this connection is not always exploited by school educators, parents, or science-center professionals. Research on learning (Bransford, Brown, & Cocking, 1999) indicates that knowledge obtained through a variety of contexts supports what is called "flexible transfer and fluent access." Essentially, this means that people are more likely to extract relevant elements of what they have learned and are better able to apply them in new settings. However, the transfer must be activated in some fashion. Often the activator is a teacher, a parent, or a science center staff member who draws the learner's attention to a relevant item. High-quality exhibits can also elicit this transfer.

An example may make the point better. One visitor interviewed by the author shared this story:

> *I had learned the Pythagorean Theorem in school. I knew the formula* $A^2 + B^2 = C^2$*, but I didn't get it until I was playing with a science center geoboard. The staff person suggested I make a right-angle triangle with a rubber band, then for each side make a square with the triangle line being the base of the square. So I made three squares—one on each side of the triangle. The staff person said: Count the number of little squares included within each big square. I did and when I realized the number of squares in the two squares on the sides equaled the number of little squares (in the square) on the third side—I realized what the formula meant. I then reworked the original triangle and repeated the process. I discovered the Pythagorean Theorem all over again.*

Observations made in the case studies support our assertion that science centers have the capacity for contributing to improved knowledge and understanding of mathematics—even as the need for additional staff development for science center professionals is acknowledged. Science centers can provide real-world connections for mathematics and help answer questions such as "Why do we need to do algebra?" The site visit teams concluded that because science centers are hands-on, they can support the formal instruction provided by schools. By working together, conceptual understanding of mathematics is enhanced. Mediated mathematics experiences—initiated by a helpful person, an engaging exhibit, or useful and fun activities— can contribute to reversing negative or "can't do" attitudes found among children and adults.

Linking exhibits and programs to NCTM and/or state Standards makes the science center a valuable resource for the K–12 school population.

Across the nation, state departments of education are calling for standards-based education that is coupled with tests of varying significance for K–12 students. The use of Standards in designing exhibits, programs, and materials can be a benefit to the teachers in schools and to parents wishing to support mathematics learning of their children.

In this study, half of the science centers generated their exhibits, programs, or materials based on either the NCTM Standards or the state mathematics standards. That is, when considering what to create, they went to the standards first and designed around the concepts. This approach was used by Pacific Science Center to construct the exhibits and the lessons for the *Mathfinder* van and by Ann Arbor to develop the *Geometry* exhibition. It was also part of the *EQUALS Investigations* development process.

Alternatively, science centers can link exhibits, programs, and other materials, such as kits or web sites, to the Standards after they are built. This approach will help illuminate for the visitor what elements of mathematics are being addressed. It also tends to result in a broader diversity of exhibits and a less targeted focus.

Either approach gives the visitor the benefit of knowing where the learning and experiences fit in with a defined core of knowledge. The intentional use of Standards gives the staff a focusing device and seems to result in multiple and deeper explorations of a given concept. This is precisely opposite from what is often found in American mathematics curricula, which continue to be "a mile-wide and an inch deep" [24] (1997).

Highest-quality exhibits, programs, and materials come from "intentionality" — where the products are designed in accordance with a particular philosophical or theoretical position and with commitment to achieving the intended outcome.

The intention and philosophical orientation of the science center make a difference in its approach to doing mathematics. While this may seem obvious, it was discovered that the results were more subtle and complex than this statement suggests at first reading.

Most importantly, where the exhibit and program quality in mathematics was highest, it was found that there was an intentional focus on what the visitor was learning, as opposed to what the exhibit or program was "teaching." In other words, exhibit or curriculum designers appeared to evaluate their products with questions such as "What does the visitor learn through this experience?" Looking not only at what visitors do, but also at what they seem to understand allows the designer to refine the exhibits or materials until what the learner does and understands matches the intent of the museum educator and the designer more closely.

[24] This phrase was used in the curriculum analysis provided for the Third International Mathematics and Science Study (TIMSS).

There is a parallel orientation in some schools that are working toward standards-based education. That is, by focusing on what children learn, teachers are learning to improve their teaching strategies and children are making greater progress[25]. It is clear that in the science-center field, the museum can find a niche in which it contributes to the community and its work in a constant refinement of that role. For example, Lawrence Hall's focus on equity has caused the science center to grapple with equity issues through multiple efforts. With each new program offering, they refine their understanding of what it takes to achieve equity. Mathematics programming was one more step toward equity. Likewise, the Exploratorium grapples with notions of inquiry, and Fort Worth wrestles with what it means to construct "extraordinary learning environments."

A substantive philosophical framework gives science center staff a touchstone to evaluate the work they do. It is also likely to draw visitors because people develop expectations about the nature of the experiences they will encounter. A framework counters perceptions of superficial amusement, increases depth of experience, and enhances the credibility of the institution with funders looking for more than entertainment values.

The human resource—the staff and its advisors—is the science center's most valuable asset for engaging in mathematics conversations and experiences.

The depth of talent and experience of the staff and advisors made a difference in the quality of mathematics exhibits, programs, and materials offered. This finding may correspond to the earlier finding about product quality, perhaps in part because the intentionality—or corporate culture—of the science center is learned.

The expertise, intellectual wisdom, and artistic and technical talent of science center professionals combine to make exhibits and other science center programs and materials credible and interesting. Successful science centers hire and retain staff whose values and experiences match the values and focus of the museum. At the same time, these effective science centers select projects that best utilize their staffing talent.

Alternatively, science centers sometimes decide to do a program or project and assign staff based on availability, rather than the preparation that individuals bring to the area. Although this may be necessary because of employment practices or other concerns, the findings from learning research suggest an issue to consider here: Expert knowledge comes with organized conceptual schemas that guide how problems are represented and understood within the domain (Bransford,et.al, 1999). Furthermore, different domains require different approaches for learning. The implications for science centers doing mathematics mean that they must address the staffing or design-team issue.

The talent question was addressed in several ways, including acquiring the necessary expertise by relying on a team of talent used as advisory personnel, such as outside mathematicians or mathematics educators, perhaps through short-term contracts with them. A note of caution, however—the content oversight for mathematics must be sustained and have a strong voice in the exhibits, programs, and materials design in order

[25] In a 1999 conversation with Greg Hall, the former Assistant Superintendent for Assessment in Alberta, Canada, Hall attributed the success of Alberta schools in mathematics on the TIMSS assessment to adopting and implementing the NCTM Standards.

to work. When it is not present, the design can slip away from content accuracy, away from the intended focus, and even toward unintended misconceptions.

Having credible content expertise is critical, as is having expertise in learning. A knowledge of pedagogy is important, especially in crafting curriculum. The most successful exhibits seemed to reflect a kind of knowledge the developer has that may be akin to Shulman's pedagogical content knowledge[26] (Ibid). That is, good exhibit design accounts for the ways people engage exhibits and learn from phenomena and seems to precipitate actions that are likely to activate connections among concepts. Might there be a techno-pedagogy that underlies the technology of exhibit design? Designers who applied technical and mechanical expertise with an artist's perspective help to create attractive and enticing exhibits.

Finally, for science centers, critical friends are essential. Either uninvolved staff or invited outsiders are needed to review exhibits. Where this practice occurred with mathematics exhibits and programs, the result was visitor or user experiences that more closely matched the intent. When this type of feedback is overlooked, exhibits, programs, and materials may miss the target and may even create misconceptions in the user. Sometimes the designers whose ideas are being tested may be unable to see how the exhibits, programs, and materials are being used because their intentions cloud their perceptions.

In general, the site visits revealed these insights:

- Competitive salaries and benefits are important for reducing turnover and the need to retrain constantly.
- Providing time to think and reflect allows staff to create better exhibits, programs, and materials.
- Encouraging professional growth makes a difference in keeping staff, sustaining their enthusiasm, and providing a stimulus for improving exhibits, programs, and materials.

Science centers choose to do mathematics exhibits, programs, and develop materials based on what they already do well.

The most effective outcomes happened when the science center grew the exhibits, programs, and materials out of their own strengths. For example, Pacific Science Center already had a statewide van program with operational funding. To suggest a mathematics-focused van meant they could rely on a known and successful delivery

[26] Pedagogical content knowledge requires that the teacher own the content knowledge of a discipline in a way that includes information about barriers or difficulties a learner must overcome to understand the concepts. According to Shulman, it is not sufficient to know the content and generic teaching strategies, since different disciplines require different strategies in order for students to construct meaningful understanding.

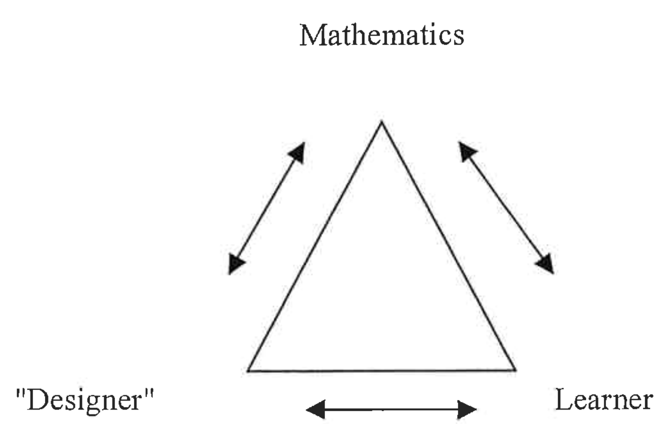
format and concentrate on transitions to a new content area. Also, it is appropriate that science centers start small and learn from the experience.

Exhibits, programs, and materials are more successful and effective when relationships are built with end users as partners who are equal contributors to the outcomes.

Mark St. John, of Inverness Research Associates, provided Fort Worth with a framework for considering the relationship of the science center with the content it provides and the visitors who use it. Fort Worth has utilized this framework in developing programs and exhibits[27]. This technique is a useful tool when thinking about the appropriate involvement of others during the design process and in thinking about the potential role for science centers in K–12 mathematics.

Begin with the triangle of relationships shown below. The arrows reflect the intellectual relationship the learner must develop with the content, the expert knowledge the designer has, and the pedagogical knowledge that connects the designer (of either exhibits or experiences) with the learner. The connecting arrows are the critical element here, because they reflect the process of creating, defining, and nurturing the relationships.

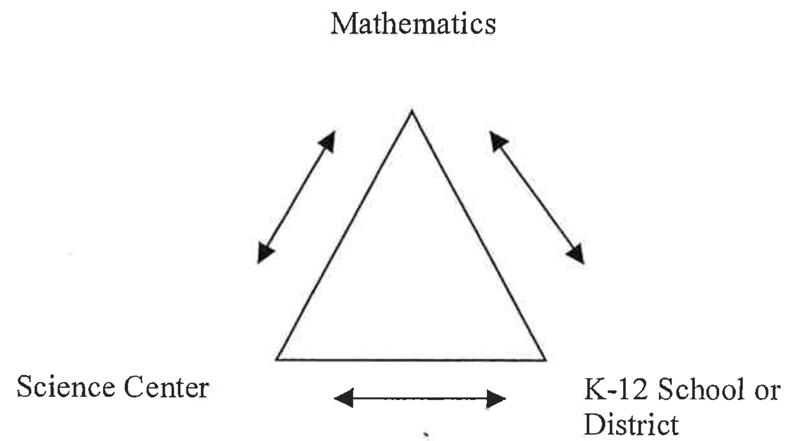
Framework of Relationship



The site-visit team discovered in its work that there is an extension of the idea of relationships in the model above. From talking with the different science centers, the

[27] From Mark St. John, in conversation with Fort Worth staff.

team discovered that the entities at the triangle vertices could change and could be used to define other important relationships. For example, in the mathematics triangle, substitute "the K-12 education system" for "learner" and "the science center" for "designer," since it is important to think of the science center as existing within a complex system of relationships. This gives an additional perspective for placing mathematics within science centers.



Once again the connecting arrows point to the relationships (and partnerships) that invigorate the exhibits, programs, and materials. The team found that the more successful mathematics programs came from institutions in which more than cursory attention was paid to the process of partnership. These institutions were better able to meet the needs of the audience, and the relationships further defined and secured a particular niche for the science center.

Paying attention to the connections between the science center, the K–12 system, and mathematics helps to sustain the science center's presence in the minds of parents, teachers, and community members, as well as of funders—be they government, corporate, or philanthropic.

Although the potential is enormous, there is almost no research or evaluation of mathematics learning in science center exhibits, programs, or materials.

Generally, science centers do not conduct research on the design or outcomes of their exhibits or programs. Often funders will request an evaluation, but frequently the data are not published in a manner that serves the field. Despite asking at each site, the team was unable to obtain any significant research or evaluation findings from the science centers visited. The science centers did share anecdotal stories about what visitors did and said about exhibits or programs and occasionally about the benefits accrued from the experiences.

Emerging Issues

A lot of questions surfaced during the study and stimulated long, thoughtful discussions. Some of the questions, responses, and insights shared during the study are presented below. These are some of the issues to be addressed, should science centers proceed with a mathematics agenda.

How to choose topics?

The TIMSS reports gives one measure of information about what content is needed. A review of the TIMSS findings (*A Sourcebook of 4th-Grade Findings: TIMSS*, 1997) showed that U.S. 4th grade students had more difficulty on measurement, estimation, and number sense than their international counterparts. In 8th grade (*A Sourcebook of 8th-Grade Findings: TIMSS*, 1997) Beaton et al., 1996) students scored lower on geometry, measurement, and proportionality when compared to other nations. At the 12th grade level (*Mathematics and Science Literacy in the Final Year of Secondary School*, 1998), the TIMSS items were selected for "mathematics literacy" and represented multiple domains. The design and reporting of achievement scores makes specifying content challenges difficult, and since at the 12th grade, students performed poorly on the exam it makes sense to focus on the content areas of previous known trouble.

Most of the science centers visited were already addressing several of the most difficult sections of the TIMSS test. For example, Ann Arbor developed a geometry exhibit and Pacific Science Center targeted measurement in its van programs. However, during the discussions with educators, the team found a strong need for support in the area of number sense—specifically, helping children develop solid understanding of multiplication, division, and, most important, fractions and decimals. Frequently, the achievement failures do not *begin* at middle school, but rather it is at middle school that the lack of understanding becomes apparent.

Local school districts and/or state education offices also have needs to be met in mathematics. Typically, schools have annual tests that describe the achievement of students. The results from these tests and information from districts can provide deeper details about what is needed at a local level.

There is an interesting point to consider regarding topics. Asking people what they *want* is an important approach; considering what people *need* may set science centers onto a different pathway. People need higher-order mathematics, but science centers reported that parents want remediation (i.e., drill and practice) for their children.

Should we address mathematics directly?

Choosing to do mathematics with science center exhibits may mean reviewing extant exhibits, programs, and materials and unpacking the inherent mathematics. The Exploratorium is thinking along these lines. Or it may mean choosing to do an explicit mathematics exhibition, where each component targets a particular concept, as in Ann

Arbor's *Geometry* exhibition. Or, it might mean deciding that all future exhibitions will address both science and mathematics, as they typically work together. This approach is the direction Fort Worth intends.

What is the role of NCTM or state Standards?

Content choices might be considered in light of either the NCTM Standards or the state's Standards or frameworks. A free-choice, informal learning institution is not obligated to follow structures that schools follow. However, doing so ensures that two particular audiences—teachers and parents—will have a powerful reason to pay attention and to bring schoolchildren to the museum. Obviously, teachers would find such support advantageous, especially in this time of high-stakes tests. Parents can also support their children's learning of mathematics when good, interactive exhibits help illuminate concepts.

As noted above, the National or State Mathematics Standards can provide a framework for choosing appropriate content for exhibits. They can also be used to entice new audiences to the science center, such as teachers of mathematics or parents seeking to improve their children's achievement in mathematics. There is another reason for using the Standards to frame exhibits or other science center exhibits, programs, and materials—that is, to deepen the mathematical literacy of the general public.

The Standards define what a mathematically competent individual needs to know and be able to do. Children are not the only ones who need to attain higher levels of performance, as reflected by the Standards. In this rapidly changing economy, which relies increasingly on technology, technical competence, and extraordinary problem-solving capacity, many adults find their own knowledge base in mathematics quite limited. Unfortunately, there are few places or mechanisms by which to learn mathematics informally. Lay people get access to significant amounts of science interpretation through science centers, television, radio, newspapers, and magazines. There is nothing parallel for mathematics.

What does learning theory suggest?

Another factor influencing content choice comes from what is understood about learning. Developmentally, people engage ideas—make meaning—first through action, then with images (imagination), and finally through symbolic manipulations (Reynolds, 2000). Experiences support the developmental sequence of how people engage with ideas. According to Sherry Reynolds:

> *Piaget noted...the advantage of the development of cognition is that it frees us to think without having to actually do things..... This ability is potential, in the sense that it does not develop unless the person has had opportunities to engage in that kind of thought and to attempt to solve problems that will develop it.*

Thus the areas of most (academic) challenge may be the content areas that require more active manipulation and application of imagination. In other words, if learners are challenged in understanding proportionality, then what might be needed are hands-on exhibits in this domain that would support concept development.

How do science centers get and keep visitors interested?

The team found that more sophisticated mathematics engaged science center visitors longer in solving problems, provided the exhibit required interaction from the visitor. Complex mathematics explaining a visual phenomenon resulted in short, almost nonexistent time on task. Where complex ideas were presented so that visitors could manipulate the variables, visitors spent significant amounts of time. For example, computer simulations with fractals[28] engaged visitors for significant time periods.

Getting the visitor to notice or pay attention to exhibits can be challenging, particularly as some visitors—mostly children—race from object to object. Ultimately the visitor settles down and shifts from context building— getting the lay of the land—to more sustained investigation with something that catches his or her attention. Humans constantly and simultaneously process incoming sensory information. What triggers attention is typically something new or something unexpected. This is the novelty effect.

The novelty effect can happen when something familiar includes a new dimension. For example, there were multiple variations of the Normal Distribution exhibit in several science centers. However, in a side visit to Dallas' Science Place, the team noted this same exhibit designed with an additional element. It used table tennis balls and glass pegs, so that as the balls dropped a musical sound was produced. Around this exhibit there always seemed to be a crowd of visitors who watched through more than one cycle. The exhibits seemed to attract more visitors than similar examples observed elsewhere.

What should we call it?

One of the interesting, unanswered questions from this study is whether to inform visitors at the beginning that they are doing mathematics. Using a friendly title may entice visitors into trying the activities. However, a major problem in mathematics today is that people have not linked activities or experiences in mathematics with traditionally understood terminology for specific concepts. This leads to confusion and misunderstanding. For example, in Washington State the EALRs used the term "Spatial Sense," but parents forced the framework developers to revert to the more commonly understood "Geometry," because the selected terminology did not convey meaning to the adults.

The familiar and iconic may convey meaning in a way eye-catching titles may not. However, initial reaction by many public visitors suggests that mathematics terms evoke

[28] The author observed the fractals exhibit at Paris' La Cite des Sciences et de l'Industrie, which was not part of the study, but was visited during the time frame for the study.

unpleasant memories and result in people's avoiding the exhibition. Might market research better inform this decision? Would a public relations campaign generate more enthusiasm for mathematics language?

What would it mean if science centers did mathematics the way they do science?

If one looks at the 4th grade science achievement on the Third International Mathematics and Science Study (TIMSS), U.S. students scored in the top third of nations (above average) in the international comparison. In contrast to science success, mathematics in the United States has not fared as well. The findings showed that U.S. students are average in mathematics during the elementary years, but decline significantly in achievement by the end of high school.

Some researchers have argued that NSF's substantial investment in elementary science education and informal science institutions' strong focus in supporting elementary teachers and students in science learning are factors contributing to science success.

Crediting science centers with even a small piece of the science achievement on TIMSS opens the door to this question: What if science centers offered more exhibits, programs, workshops, and publications in mathematics—perhaps equal in number and emphasis to their science offerings? Would there be greater achievement in mathematics?

Recommendations

The United States needs and wants improvement in mathematics accomplishments. Achievement has benefited from the introduction of Standards, but the goal is still out of reach. Science centers, as part of the educational infrastructure (St.John, 1996) can make a significant contribution by providing informal mathematics learning experiences.

The primary recommendation from the team for ASTC is that the organization undertake a capacity-building initiative that would enable science centers to offer more and better mathematics in more institutions nationwide.

Should ASTC choose to further mathematics development in the science center field, an initiative might include the following elements:

1. A showcase (conference or workshop) of the current, best mathematics exhibits, programs, and materials in science centers.

The team has identified some, but not all of the mathematics exhibits, programs and materials currently in the field. There are others. To encourage museums to take up more mathematics in their institutions, it is recommended that a hierarchy of steps running from least formidable to more complicated be employed. Some of the steps might be:

- Organizing in-house discussions about mathematics, with assistance from local mathematics educators
- Using or copying a mathematics-based exhibit from another institution
- Adding mathematics to exhibit text for existing science exhibits
- Offering *FAMILY MATH* type classes
- Including a few mathematics-based exhibits when conceiving a new science exhibition
- Developing mathematics-based exhibition programs
- Offering after-school classes or adding a mathematics segment to regular after-school science classes
- Sponsoring teacher workshops with others as leaders
- Offering short teacher workshops
- Seeking funds for more extensive work with teachers.

There appears to be sufficient interest to hold a working conference that would allow professionals to examine the exhibits, programs, and materials available, to discuss some of the questions raised by this study, and to give science center employees an opportunity to imagine and create new products. At the same time, funders could be invited to the showcase to excite them about the possibilities. A number of federal agencies, foundations, and corporations have already demonstrated interest in improving the state of mathematics literacy in the United States. In addition, foundations that consistently express concern for children and their learning experiences may find that new avenues for improving mathematics understanding constitute a desirable project.

2. Staff development for science-center professionals seeking to include mathematics in their exhibitions, program offerings, or materials.

Part of the current dilemma is that science centers do not have many mathematics educators, mathematics enthusiasts, or mathematicians on staff. Learning mathematics, as portrayed by the NCTM Standards, will be an important first step. Several strategies appear tenable. A series of institutes, such as those offered by the Teacher Educators Network[29] during the early 1990s, would deepen the capacity of educators and exhibit designers to think about mathematics.

Alternatively, sending staff on extended site visits to work with those already engaged in doing mathematics has a powerful learning value, as this SGER-supported project has demonstrated. Finally, although ASTC conferences already include sessions on mathematics, mostly as exhibit or program components, it might be productive to include a few sessions where mathematics is explored explicitly.

3. Creating working relationships with the National Council of Teachers of Mathematics and the Mathematical Association of America.

ASTC has formed many successful, fruitful partnerships in the past. Although individual science centers can and do build relationships with local teachers groups, there is a powerful statement made when national organizations agree to collaborate. Furthermore, by building the relationship, the organizations create fertile ground for new ideas.

4. Creating a presence for mathematics by inviting mathematicians or mathematics educators to address the field.

Nationwide, there are interesting, exciting, and easy-to-understand mathematicians who are seeking to improve public understanding of mathematics. Individuals who might be considered include authors whose works have stimulated public interest in mathematics or have addressed the all-too-common fear of mathematics. In addition, the mathematics education community can be a rich source of people who are passionate about mathematics and learning.

[29] A grant from NSF in 1990 provided funds to ASTC's Teacher Educators Network (TEN) to conduct three inquiry institutes for science center–based teacher educators and partners from university faculty and/or classroom teachers. There was also a fourth, follow-up institute.

5. Finding out if doing mathematics in science centers makes a difference in visitors' learning or attitudes about mathematics.

There has been almost no research completed that examines the effectiveness of informal learning experiences in mathematics. Children's television programming, such as "3 2 1 Contact," has looked at learning outcomes and been able to report success. However, since few science centers have done mathematics, and fewer still have completed evaluations of exhibits, programs, and materials, this is a natural area for research.

6. Reaching out to universities, colleges, education organizations, and the K–12 school system as partners in achieving improved mathematics understanding by children and adults.

In exploring the world of mathematics education, a number of institutions proved to be very eager to work with science centers. However, as noted before, it boils down to building relationships. For example, hosting initial meetings, agreeing to explore ideas, fostering trust and patience, and negotiating the rough waters experienced at the confluence of two or more cultures are all part of collaborating to achieve an outcome.

For help in brokering these relationships it is suggested that science centers connect with their regional Eisenhower Consortium for Mathematics and Science. While many consortia members have reached out to the informal science community, for this SGER project the Eisenhower Consortium for Mathematics and Science Education at SERVE (South Eastern Regional Vision of Education) demonstrated how the Consortium's brokering role may be used to further a mathematics effort between science centers and schools. SERVE identified people, provided resources, and contributed mathematics expertise for at least one site visit and for the initiating meeting held in Edmonton, Alberta, in October 1998.

References

Beaton, A., Mullis, I., Martin, M., Gonzalez, E., Kelly, D., & Smith, T. 1996. *Mathematics Achievement in the Middle School Years*. Chesnut Hill, MA: TIMSS International Study Center.

Bransford, J. D., Brown, A. L., & Cocking, R. R. 1999. *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academy Press.

Devlin, K. 1998. *Life by the Numbers*. New York: John Wiley & Sons, Inc.

Devlin, K. 2000. *The Math Gene: How Mathematical Thinking Evolved and Why Numbers Are Like Gossip*: Basic Books.

Eighth-Grade Mathematics Lessons: United States, Japan, and Germany [video]. 1997. Washington, D.C.: U.S. Department of Education, Office on Educational Research and Improvement.

Enzensberger, H. M. 1997. *The Number Devil: A Mathematical Adventure*. New York, NY: Metropolitan Books, Henry Holt and Company, LLC.

Leitman, R., Binns, K., & Unni, A. 1995. "Uninformed Decisions: A Survey of Children and Parents about Math and Science." In *NACME Research Letter, 5*(1).

Mathematics and Science Literacy in the Final Year of Secondary School. (1998).

Reynolds, S. 2000. *Learning Is a Verb: The Psychology of Teaching and Learning*. 1st ed. Scottsdale: Holcomb Hathaway.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. 1997. *A Splintered Vision: An Analysis of U.S. Mathematics and Science Curricula.*

A Sourcebook of 4th-Grade Findings: TIMSS. 1997. Philadelphia: Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education.

A Sourcebook of 8th-Grade Findings: TIMSS. 1997. Philadelphia: Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education.

Sprenger, M. 1999. *Learning & Memory: The Brain in Action*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

St. John, M. 1996. *An Invisible Infrastructure: Institutions of Informal Science Education*. Washington D.C.: Association of Science-Technology Centers.

Wolfe, P. 1996. "Translating brain research into practice." Paper presented at the ASCD Annual Conference.

Appendix

The Principles and Standards For School Mathematics

National Council of Teachers of Mathematics

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1 **Number and Operation** **1. Understand numbers, ways of representing numbers, relationships among numbers and number systems**	count fluently with understanding and recognize "how many" in small sets of objects	understand the structure of the base ten number system, including representations of decimals	work flexibly with equivalent fractions, decimals, percents; compare and order these numbers efficiently and accurately, find their approximate locations on a number line, and choose appropriate and convenient forms of these numbers for solving problems	increase their understanding of systems for representing numbers and quantities, including matrix representations for arrays of quantities
	understand the cardinal and ordinal meaning of numbers in quantifying, measuring, and identifying the order of objects	read and write large numbers, and relate notation to the meaning of these numbers	develop meaning for integers, and be able to represent, compare and order them	compare and contrast properties of numbers and number systems
	connect number words, the quantities they represent, numerals, and written words, and represent numerical situations with each of these	develop meaning for fractions as parts of a unit whole, as a part of a collection, as numbers, and as a division of whole numbers	develop an understanding of large numbers, including the use of benchmarks to comprehend their magnitude; and recognize understand and appropriately use various representations for large numbers (e.g., exponential, scientific, and calculator notation)	begin to understand complex numbers as a superset of the real numbers and as a system containing solutions for equations that not solvable over the real numbers
	develop an understanding of the relative magnitude of numbers and make connections between the size of cardinal numbers and the counting sequence	continue to develop a sense of the relative magnitude of numbers, with a focus on powers of 10 (esp. hundreds and thousands) and their role as benchmarks in the number system	use number theory concepts (e.g., factors, multiple, prime factorization, relatively prime numbers) to solve problems and to understand ideas about rational numbers	become familiar with finite sequences and series, including arithmetic and geometric examples, and develop an informal understanding of some infinite sequences and series, especially geometric series
	develop an understanding of the multiple relationships among whole numbers by comparing, ordering, estimating, composing, decomposing, and grouping numbers, including beginning understandings of place value	develop meaning for factors and multiples, become fluent with factor and multiple relationships, and classify numbers according to their factor (prime, composite)	develop an understanding of the properties of the integer and rational number systems (e.g., order, density, and additive and multiplicative inverses)	
	understand and represent familiar fractions, such as 1/2 and 3/4	read and write fractions and decimals and relate the notation to the meaning of these numbers	recognize and use commonly encountered irrational numbers (e.g., Π)	

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1.1 continued		recognize and use common fraction, decimal, and percent equivalents		
		identify characteristics of particular numbers (e.g., odd/even, prime/composite) and use this information to describe characteristics of classes of number (e.g., multiples of 7)		
		develop strategies for judging the size of fractions and decimals and for comparing them, using a variety of models and of benchmarks (such as 1/2 or .5)		
STANDARD 1 **Number and Operation** **2. Understand the meaning of operations and how they relate to each other**	understand different meanings of addition and subtraction of whole numbers and the relation between the two operations	understand the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array)	extend understanding of operations to include operations on fractions, decimals, percents, integers, and nonnegative whole number exponents	develop an understanding of the meaning of, and representations for, operations on vectors and matrices, and, with appropriate technology, be able to use these operations to solve systems of linear equations
	understand situations that lead to multiplication and division such as equal groupings of objects and sharing equally	identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division)	understand the effects of operating among fractions, decimals, percents, and integers	develop fluency operating on real and complex numbers, vectors and matrices, using by-hand operations for simple cases and using technology for more complex cases
	develop understanding about the effects of the operations on whole numbers	identify and use properties of operations to solve problems (e.g., 28 x 7 is equivalent to (7 x 20) + (7 x 8)	recognize and use the properties of operations on integers and other rationals, such as closure, associative, commutative, and distributive properties understand and use the inverse relationships of addition and subtraction, multiplication and division, and square and square roots to solve problems extend understanding of counting to include elementary combinatorics	continue to develop an understanding of permutations and combinations as counting techniques in increasingly complex situations

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1 **Number and Operation** **3. Use computational tools and strategies fluently and estimate appropriately**	develop and use strategies and algorithms to solve number problems	develop fluency with single-digit multiplication facts and their related division facts by grade 4, and use facts to efficiently compute related problems (e.g., 30 x 50 is related to 3 x 5, 300 x 5 and 15 x 100)	develop, analyze and compare algorithms for computing with fractions, decimals, percents and integers, and become efficient and accurate in computing with them	analyze algorithms for operations with numbers, recognize some of the roles and limitations of particular algorithms, and be able to verify the viability of selected algorithms
	develop fluency with addition and subtraction facts by the end of second grade	develop and compare whole-number computational algorithms for each operation that are based on understanding number relationship, the base ten number system, and the properties of these operations, and by the end of grade 5 develop efficiency and accuracy in using algorithms	develop, analyze and compare methods for solving problems, involving proportions (e.g., scaling, finding equivalent ratios)	develop an understanding of the effects of measurement error on computed values
	compute using a variety of methods, including mental computation, paper and pencil, and calculators, and choose an appropriate method for the situation	develop and use computational estimation strategies based on understanding of number concepts properties and relationships	develop and refine strategies for estimating (including fractional quantities); use estimation as a means to check the reasonableness of results	develop the ability to distinguish between estimation and approximation, and use each appropriately in technological and nontechnological settings
	recognize whether numerical solutions are reasonable.	estimate sums and differences of common fractions and decimals using benchmarks (e.g., 3/8 + 1/3 must be less than 1 since both fractions are less than 1/2)	select and use appropriate methods for computing from among mental arithmetic, estimation, paper and pencil, and calculator, depending upon the situation at hand develop and use visual models, benchmarks and equivalents to add and subtract with common fractions (e.g., 1/2 + 1/4 is the same as 1/4 + 1/4 + 1/4 or 3/4; or parts of a "pie")	

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1.3 continued			develop and use visual models, benchmarks and equivalents to develop computations, procedures for addition and subtraction of decimals (e.g., determining 0.10 + 0.15 by shading in a 1- by 10 grid) choose appropriate computational procedures and tools (e.g., calculators, pencil/paper, mental computation) to solve problems.	
STANDARD 2 **Patterns, Functions, and Algebra** **1. Understand various types of patterns and functional relationship**	sort and classify objects by different properties	identify, describe and extend geometric and numeric patterns, including growing and shrinking patterns	analyze, create, and generalize numeric and visual patterns, paying particular attention to patterns that have a recursive nature	recognize equivalent forms of an expression, equation, function, or relation
	order objects by size or other numerical property (seriation)	represent and record patterns, using tools such as tables and graphs	use patterns to solve mathematical and applied problems	be familiar with classes of functions, including linear, quadratic, power, polynomial, rational, absolute value, exponential, logarithmic, trigonometric, and step functions; understand piecewise-defined functions and their properties; analyze the effects of parameter changes; and describe local and global behavior
	identify, analyze and extend patterns and recognize the same pattern in different manifestations	identify and describe relationships between two quantities that vary together (e.g., the length of a square and its area)	represent a variety of relations and functions with table, graphs, verbal rules, and when possible, symbolic rules	select appropriate representations (numerical, graphical, verbal, and symbolic) for the functions and relations embedded in quantitative situations, convert flexibly among representations, interpret representations, and use them to interpret the situations represented

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 2.1 continued	describe how both repeating and growing patterns are generated	investigate and describe situations involving inverse relationships (e.g., the more friends, the fewer the cookies for each person, or the larger the denominator in a unit fraction, the small the quantity)		use a variety of symbolic representations, including recursive definitions and parametric equations, to explore the behavior of functions and relations
		identify, express, and verify generalizations and use them to make predictions (e.g., doubling a number, then doubling again is the same as multiplying by four)		reason (from graphs, tables, and formulas) about functions derived from other functions via transformation (e.g., g(x)= f)x-2)+5), inversions, composition, and arithmetic combination.
STANDARD 2 **Patterns, Functions, and Algebra** **2. Use symbolic forms to represent and analyze mathematical situations and structures**	illustrate general principles (e.g., commutativity) using specific numbers	identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division)	develop a sound conceptual understanding of equation and of variable	represent situations that involve variable quantities with expressions, equations, inequalities, and systems of equations using a variety of equivalent forms
	understand reversal of operation	identify and use algebraic properties of operations to solve problems (e.g., 28 x 7 is equivalent to (7 x 20) + (7 x 8)	explore relationships between symbolic expressions and graphs, paying particular attention to the horizontal and vertical intercepts, points of intersection, and slope (for linear relations)	develop fluency operating on polynomials, vectors, and matrices using by-hand operations for the simple cases and using technology for more complex cases
	use concrete, pictorial, and verbal representations of numerical situations, including invented notation	develop the concept of variable as a useful tool for representing unknown quantities	become fluent in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities	understand symbolic algebra as abstracted arithmetic
	use appropriate symbolic representation of mathematical situations	use variables (boxes, letters, or other symbols) to solve problems or to describe general rules	use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships	be able to explain, compare, and contrast the major properties of the objects and operations defines within and across systems (e.g., rational numbers, polynomials, matrices, and functions) as they follow certain rules or laws of structure

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 2.2 continued				develop strategies for deciding whether symbolic results generated with technological tools are reasonable, and interpret such results in meaningful ways
STANDARD 2 **Patterns, Functions, and Algebra** **3. Use mathematical models and analyze change in both real and abstract contexts**	make comparisons and describe change qualitatively (e.g., taller than)	represent and investigate how a change in one variable relates to the change in a second variable (e.g., the height of a plant over time)	model and solve contextualized problems using various representations, such as graphs and tables, to understand the purpose and utility of each representation	model a wide range of phenomena with a variety of functions, including linear, quadratic, exponential, rational, trigonometric, and recursively defined functions, and recognize that a particular type of function can model many different situations
	make comparisons and describe change quantitatively (e.g., 3 inches taller)	identify and describe situations with varying rates of change (e.g., a fund-raising effort brought in a small, steady amount of money in the beginning, but more each day as the deadline approached)	develop an initial understanding of rate of change, with emphasis on the connections among slope of a line, constant rate of change, and meaning in context	approximate and interpret accumulation and rates of change, both graphically and numerically, for functions representing a variety of situations.
	model concrete situations using addition and subtraction of whole numbers		explore different types of change occurring in discrete patterns, such as proportional and linear change	approximate and find intercepts, local extreme values, and asymptotic behavior of functions, and interpret such results in given contexts
STANDARD 3 **Geometry and Spatial Sense** **1. Analyze characteristics and properties of two- and three-dimensional geometric objects**	recognize, name, build, draw, describe, compare, and sort two- and three-dimensional shapes	identify, compare, and analyze attributes of two-and three-dimensional geometric figures	precisely describe, classify and compare types of plane and solid figures (e.g., angles, triangles, quadrilaterals, cylinders, cones, etc.) according to their main features	explore relationships among, make and test conjectures about, and solve problems involving classes of two- and three-dimensional geometric objects
	recognize and locate geometric shapes and structures in the world	classify two- and three-dimensional shapes according to their attributes, and develop definitions of classes of shapes (e.g., triangle, pyramid)	analyze and understand geometric relationships among two-dimensional and three-dimensional figures	connect geometry to other strands of mathematics (e.g., measurement, algebra, trigonometry), relate it to other areas of interest (e.g., art, architecture), and use it to solve problems

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 3.1 continued	describe attributes and parts of two- and three-dimensional shapes	investigate, describe, and reason about the results of subdividing, combining, and transforming shapes using models and representations	use proportions to examine relationships between similar plane figures	recognize geometry as an example of a deductive system, built from undefined terms, axioms, definitions, and theorems; use deduction to establish the validity of geometric conjectures and to prove theorems
	investigate and predict the results of putting together and taking apart shapes	investigate and reason about properties of geometric figures (e.g., the number of diagonals of a regular polygon)	create and critique inductive and deductive arguments concerning geometric ideas and relationships	
	recognize congruent and similar shapes	build and use geometric vocabulary to describe two- and three-dimensional shapes	recognize and apply geometric ideas and relationships outside the mathematics classroom, in areas such as art, science, and everyday life	
	relate geometric ideas to number and measurement ideas	understand and demonstrate congruence		
		make conjectures about properties and relationships of shapes and use spatial reasoning and geometric tools to test them		
STANDARD 3 **Geometry and Spatial Sense** **Select and use different** **2. representational systems, including coordinate geometry and theory graph**	describe, name, interpret, and apply ideas of relative position in space	describe location and movement using common language and geometric vocabulary (i.e., location of an object in a room or on a grid)	learn to use coordinate geometry to display geometric relationships between related quantities, particularly when the relationship is linear	investigate and verify conjectures and solve problems involving two- and three-dimensional figures represented with rectangular coordinates
	describe, name, interpret, and apply ideas of direction and distance in navigating space	set up a coordinate graph (include axes, origin, and scale), and use it to mark and read position using coordinate pairs	use coordinate geometry to represent and examine the properties of geometric figures, especially those containing sets of parallel or perpendicular lines	explore other coordinate systems (e.g., navigational, polar, spherical) and their uses
	find and name locations with simple relations (e.g., near to) and coordinate systems (maps)	make and use coordinate maps to represent actual places (e.g., post office, school) in an area	use relationships found in right triangles (e.g., the Pythagorean relation, isosceles right triangles, 30-60-90 degree triangles) to solve problems	explore discrete/finite geometry systems (networks) and their characteristics and applications

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 3.2 continued		represent rectangles and right triangles on a coordinate system and identify vertices with coordinates	explore the use of other representational systems, particularly networks	use trigonometric relationships to solve problems
		find distance between points along horizontal and vertical lines of a coordinate system		
		given a distance, find pairs of points on the coordinate system separated by that distance		
STANDARD 3 **Geometry and Spatial Sense** **3. Recognize the usefulness of transformations and symmetry in analyzing mathematical situations**	recognize and apply slides, flips, turns; predict the effects of transformations on shapes	predict the results of sliding, flipping, and turning two-dimensional figures	describe size position and orientation of figures under informal transformations such as flips, turn, slides, and magnification	represent translations, reflections, rotations, and dilations/contractions of objects in the plane, using sketches, coordinates, vectors, or matrices, and use these representations to gain information about the transformations
	recognize and create reflectional and rotational symmetry of two- and three- dimensional objects	describe a motion or series of motions that will show that two figures are congruent	use line and rotational symmetry to describe and classify polygons and polyhedras	extend transformations to three dimensions, to include reflectional and rotational symmetry of solids
		identify and describe line and rotational symmetry in various two-dimensional shapes	understand the concepts of congruence and similarity using transformations	understand transformations (under the operation of composition) as an algebraic system of functions
		explore symmetry in three-dimensional objects	explore the composition of transformations (e.g., successive flips in different lines)	

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 3 **Geometry and Spatial Sense** **4. Use visualization and spatial reasoning to solve problems both within and outside of mathematics**	create mental images of geometric shapes (spatial memory and spatial visualization)	identify a three-dimensional shape from a set of two-dimensional views (e.g., match a solid with its front, side, and top views	develop fluency with two-dimensional representations of three-dimensional objects	draw and interpret two- and three-dimensional objects, including those involving overlapping figures/objects and those requiring auxiliary lines
	determine and represent objects from different perspectives and points of view	visualize and sketch three-dimensional shapes in two dimensions (e.g., make views, nets, or isometric drawings)	compose and decompose two- and three-dimensional figures in order to solve problems	analyze cross-sections, truncations, and compositions/decompositions of three-dimensional objects
	recognize and describe spatial relationships	describe and draw geometric shapes from recalled mental images	use geometric models to represent and explain numerical and algebraic relationships	visualize three-dimensional objects and spaces from different perspectives
		use spatial orientation to navigate to the same point from several different starting positions		
STANDARD 4 **Measurement** **1. Understand attributes, units, and systems of measurement**	recognize the attributes of length, capacity, weight, area, and time	develop awareness of measurements as approximations, and understand how the tools used to measure affect the level of precision	select appropriate units and scale to estimate and measure angles, perimeter, area, surface area, and volume	select an appropriate unit of measurement or scale, and understand the effects of the choices that are made
	compare and order objects qualitatively by these attributes	understand the need for uniform units of measurement, and develop facility in using the common units of the English and metric systems of measurement	understand both metric and customary systems of measurement, including relationship among units of the same system	analyze how changes in the measurement of one attribute of an object relate to others, such as how the change in the radius or height of a cylinder affects the surface area or volume of the cylinder
	make and use measurements in natural situations	identify attributes such as length, area, and volume, and know the type of unit and tool needed to measure each attribute		understand rate of change as a quotient of two different measures

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 4.1 continued	develop referents for estimation	carry out simple unit conversions within a system of measurement (e.g., centimeters to meters, hours to minutes)		use successive approximations to find areas and instantaneous rates of change
	develop a sense of the unit (e.g., length, area) through estimation	explore the effect on size when objects change under simple transformations (e.g., do different shapes made from linking eight cubes have the same surface area? the same volume?		
	use tools such as rulers to measure	select and apply appropriate stand units (metric and English) and tools (e.g., rulers, grid paper, graduated cylinders) to estimate and measure length, area, volume, weight, time, temperature, and angles;	be proficient in measuring angles in plane figures	apply scaling techniques to view a problem from different perspectives, such as window changes in the graphs of functions
	measure with same size unit (nonstandard and standard)	determine the perimeter, area, and volume of shapes and solids by counting segments, square units, or cubic units	develop and use formulas for the perimeter and area of parallelograms, trapezoids, circles, and simple composite figures	use radian and degree measures
STANDARD 4 **Measurement** **2. Apply a variety of techniques, tools, and formulas for determining measurements**	use repetition of units (iteration) to measure length and area	develop, understand, and use formulas to find the perimeter and area of rectangles	select techniques and tools to measure accurately, with levels of precision appropriate to the situation	understand and apply the concepts of variance and standard deviation as measures of spread in a distribution
		estimate measurements of physical objects by comparing the objects to benchmarks (e.g., estimate the size of an angle by relating it to a right angle)	use ratios and proportions to solve problems involving scale factors	use dimensional analysis for unit conversion and to verify that expressions and equations make sense
		determine the surface area of cubes and other rectangular solids by considering each face	determine an appropriate scale and use scale drawings or models in applications	determine precision, accuracy and measurement errors; identify sources (measurement or round-off errors) and magnitudes of possible errors in a measurement setting; understand how errors propagate within computations; determine how much imprecision is reasonable for various measurements

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 4.2 continued		use map scales to measure distance between locations and make simple scale drawings using grid paper	solve formulas for determining measurements simple problems involving rates and derived measure (e.g., miles per hour)	use successive approximations to illustrate and use the formulas for the volume of a sphere, a general cylinder, and a cone
				informally apply limit concepts to further develop the concepts of area and instantaneous rate of change
				combine measurement s (e.g., length, time, mass, area, volume), using rations to produce measures such as acceleration, velocity, pressure, and density, as well as dimensionless measures such as trigonometric ratios
				combine measures (e.g. mass, acceleration, distance) using multiplication to produce measures such as force, work, and person-hours
STANDARD 5 **Data Analysis, Statistics, and Probability** **1. Pose questions and collect, organize, and represent data to answer those questions**	gather data about themselves and their surroundings to answer questions that involve multiple responses	formulate questions they want to investigate	design experiments and surveys, and consider potential sources of bias in design and data collection	design and carry out appropriate methods for gathering univariate data, both to study the distribution of a variable in one population and to compare the distributions of the same variable in two different populations
	sort and classify objects and organize data according to attributes of the objects	design data investigations to address a question	recognize types of data (e.g., categorical, count, continuous or measurement, and organize collections of data)	design appropriate methods for collecting, recording, and organizing data to obtain bivariate data in order to study the association between two variables
	represent data to convey results at a glance, using concrete objects, pictures and numbers	collect data using observations, measurement, surveys, or experiments	choose, create and utilize various graphical representations of data (line plots, bar graphs, stem-and-leaf plots, histograms, scatter plots, circle graphs, and box-and-whisker plots) appropriately and effectively	select appropriate graphical representations and numerical summaries of data

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 5.1 continued		organize data using tables and graphs (e.g., bar graph, line plot, stem-and-leaf plot, circle graph, and line graph)		understand how a change in a representation (e.g., scales on a scatterplot, categories in a two-way table, and bin size of histogram) affects the information it conveys
		use graphs to analyze data and to present information to an audience		use calculators and computer applications (e.g., spreadsheets, simulation software, and statistical software) appropriately to assist in data collection, organization, and representation
		compare data representations to determine which aspects of the data they highlight or obscure		
STANDARD 5 **Data Analysis, Statistics, and Probability** **2. Interpret data using methods of exploratory data analysis**	describe parts of the data and the data as a whole	describe the shape and important features of a set of numerical data, including its range, where the data are concentrated or sparse, and whether there are outliers	find, describe, and interpret mean, median, and mode as measures of the center of a data set; know which measure is best to use in particular situations; and understand how each does and does not represent the data	compute, identify, and interpret measures of center and spread (e.g., range, variance, and standard deviation, and interquartile range)
	identify parts of the data with special characteristics (e.g., the category with the most frequent response)	describe the center of sets of numerical data, first informally, then using the median	describe and interpret the spread of a set of data using tools such as range, interquartile range, and box-and-whiskers graphs	describe shapes of one- and two-dimensional data sets
		classify and describe categorical data (e.g., ways we travel to school) in different ways; analyze and compare the information highlighted by different classifications compare related data sets, with emphasis on the range, center, and how the data are distributed	interpret graphical representations of data, including description and discussion of the meaning of the shape and features of the graph, such as symmetry, skewness, and outliers analyze associations between variables by comparing the centers, spreads, and graphical representations of related data set	look for symmetry and skewness, clusters and gaps, and possible outliers in data and consider their effects on the interpretation of the data recognize how sample size or transformations of data affect shape, center, and spread

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 5.2 continued				
		propose and justify conclusions based on data	examine and interpret relationships between two variables, using tools such as scatter plots and approximate lines of best fit	use a variety of representations of data, including scatterplots, frequency distributions, and two-way tables
		formulate questions or hypotheses based on initial data collection, and design further studies to explore them		be able to recognize trends in bivariate data, visually and numerically, and use technology to determine how well different models (e.g., linear, exponential, and quadratic) fit data, while understanding that a perfect fit is unlikely for empirical data
STANDARD 5 **Data Analysis, Statistics, and Probability** **3. 3. Develop and evaluate inferences, predictions, and arguments that are based on data**		describe how data collection methods can impact the nature of the data set	develop conclusions about a characteristic in the population from a well-constructed sample	understand the elements involved in finding good models for phenomena
		discuss the concept of representativeness of a sample within the context of a particular example (e.g., is our class representative of other fifth-grade classes in our town? in the U.S.? in Canada? Why or why not?	through simulations, develop an understanding about when differences in data may indicate an actual difference in the populations from which the data were collected and when the difference may result from natural variation in samples	apply well-fitting models to predict unobserved outcomes
		compare the data from one sample to other samples and consider why there is variability	use data to answer the questions that were posed, understand the limitations of those answers, and pose new questions that arise from the data	evaluate conclusions based on data
		in simple experiments, infer the structure of the population through drawing repeated samples (with replacements)		use data from samples to estimate population statistics
				use and interpret the normal and binomial distributions appropriately

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 5 **Data Analysis, Statistics, and Probability** **4. Understand and apply basic notions of chance and probability**	understand notions such as certain, impossible, more likely, less likely	discuss events as likely or unlikely, and give descriptions of the degree of likelihood in informal terms (e.g., unlikely, very unlikely, certain, impossible)	make judgments about the likelihood of uncertain events, and be able to connect those judgments to percents or proportions	understand and compute probabilities of independent, disjoint, and conditional events
		estimate, describe, and test probabilities of outcomes by associating the degree of certainty with a value ranging from 0 to 1 (e.g., in simple experiments involving spinners with different fractions shaded)	understand what it means for events to be equally likely and for a game or process to be fair	understand that some phenomena are random, and apply the law of large numbers to predict long-term behavior
			compute simple probabilities using appropriate methods, such as lists, tree diagrams, or area models	use probability distributions to compute probabilities of events
			identify complementary, mutually exclusive, independent, and dependent events, and understand how these relationships affect the determination of probabilities	

THE PROCESS STANDARDS

STANDARD 6 **Problem Solving**	STANDARD 7 **Reasoning & Proof**	STANDARD 8 **Communication**	STANDARD 9 **Connections**	STANDARD 10 **Representation**
1. Build new mathematical knowledge through their work with problems	1. Recognize reasoning and proof as essential and powerful parts of mathematics	1. Organize and consolidate their mathematical thinking to communicate with others	1. Recognize and use connections among different mathematics ideas	1. Create and use representations to organize, record, and communicate mathematical ideas
2. Develop a disposition to formulate, represent, abstract and generalize in situations within and outside mathematics	2. Make and investigate mathematical conjectures	2. Express mathematical ideas coherently and clearly to peers, teachers, and others	2. Understand how mathematical ideas build on one another to produce a coherent whole	2. Develop a repertoire of mathematical representations that can by used purposefully, flexibly, and appropriately
3. Apply a wide variety of strategies to solve problems and adapt the strategies to new situations	3. Develop and evaluate mathematical arguments and proofs	3. Extend their mathematical knowledge by considering the thinking and strategies of others	3. Recognize, use, and learn about mathematics in contexts outside of mathematics	3. Use representations to model and interpret physical, social, and mathematical phenomena
4. Monitor and reflect on their mathematical thinking and solving problems	4. Select and use various types of reasoning and methods of proof as appropriate	4. Use the language of mathematics as a precise means of mathematical expression		

About the Author

Andrea V. Anderson is Research Faculty at the University of Washington, Bothell. Her two primary areas of research are teacher education and informal learning. She received her Ph.D. in Curriculum and Instruction from the University of Washington, Seattle, in 1991 and joined the Association of Science-Technology Centers as the Director of the Teacher Educators Network. Drawing upon her early years as a classroom teacher, followed by curriculum development and teacher education at the Pacific Science Center, she launched the Inquiry Institutes at Science Centers program. She also provided professional development opportunities for science center educators through a Carnegie-funded minigrant program and had oversight for the publication of a major report to the field: *An Invisible Infrastructure, Institutions of Informal Science Education.*

About the Consultant

Virginia Thompson holds a Masters degree in Mathematical Statistics from the University of California, Berkeley. She is a certified California Standard Life teacher in Elementary and Secondary Mathematics, Physics, and Biological Sciences and in Junior College Statistics and Mathematics. Virginia has taught mathematics at the elementary school and university levels. She has been a mathematical editor and coauthored *Building Confidence in Math.* Virginia is retired from the University of California, Lawrence Hall of Science, where she was the Coordinator of Public Programs/Mathematics Specialist and Acting Director of EQUALS. During her tenure at the Lawrence Hall of Science, she developed, implemented, and presented FAMILY MATH inservice programs in English and Spanish and served as the Director of FAMILY MATH.

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